Air Traffic Flow Management in the Presence of Uncertainty

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Motivation

❖ **8 significant weather elements impacting NAS**
   - Wind
   - Ceiling
   - Visibility
   - Snow
   - Freezing rain/drizzle
   - Turbulence
   - Icing
   - Thunderstorms

❖ **Each element is stochastic**

❖ **Each element has different impact on capacity**
Re-routing and holding decisions are based on predictions of the location, timing and severity of convective weather.

Deterministic TFM decisions – state of the art in terms of implementation – result in “lost” airspace capacity.

Given the history of weather forecast accuracy, there is a distinct need for TFM algorithms accounting for uncertainty.
Motivation (cont’d)

- Difficult to determine the number of aircraft to send towards an airport or airspace volume (e.g. sector) because:
  - Forecasts have significant uncertainty
  - Many flights are on the order of 3 to 6 hours
  - Uncertainty increases rapidly for forecast greater than 2 hours
  - Flow into an airport or airspace volume affected by flow into and out of all airspace volumes upstream of that airport or airspace volume

- While several TFM models exist…there are limitations due to the lack of a mapping between weather forecasts and capacity
Research Objective

- Determine “stochastic capacity” for a volume of airspace given the forecast weather and weather uncertainties

- Determine “number of aircraft” to send towards a volume of airspace given the capacity distribution for that airspace volume
Research Approach

- **Step 1:** Develop set of airspace blockage scenarios for given volume of airspace that are “consistent” with probabilistic convective weather forecast

- **Step 2:** Develop efficient (fuel-optimal) conflict-resolution algorithm

- **Step 3:** Derive “probabilistic capacity” over time using Monte Carlo simulation that combines elements of Steps 1 and 2

- **Step 4:** Determine number of aircraft to send towards volume of airspace using probabilistic capacities and two-stage stochastic program
Step 1:
Develop set of airspace blockage scenarios for given volume of airspace that are “consistent” with probabilistic convective weather forecast
Step 1: Probabilistic Forecast of Convection

1-6 hour National Convective Weather Forecast (NCWF-6)

Step1: Probabilistic Forecast of Convection (cont’d)

60-min probabilistic forecast of storms
Step 1: Modeling Approach

- **Binary cell blockage maps**
  - Passing a band limited 2D random signal (e.g. uniformly distributed random signal) defined on the work grid through a shaping filter determined by the sequence of probability matrices.
  - For each of the probability matrices, a corresponding binary cell blockage map is generated.

- **Table: Binary Cell Blockage Maps**

<table>
<thead>
<tr>
<th>Probability Matrix</th>
<th>Blockage Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>✓</td>
</tr>
<tr>
<td>0.10</td>
<td>✓</td>
</tr>
<tr>
<td>0.15</td>
<td>✓</td>
</tr>
<tr>
<td>0.20</td>
<td>✓</td>
</tr>
<tr>
<td>0.25</td>
<td>✓</td>
</tr>
<tr>
<td>0.30</td>
<td>✓</td>
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<tr>
<td>0.35</td>
<td>✓</td>
</tr>
<tr>
<td>0.40</td>
<td>✓</td>
</tr>
<tr>
<td>0.45</td>
<td>✓</td>
</tr>
<tr>
<td>0.50</td>
<td>✓</td>
</tr>
</tbody>
</table>

- **Diagram: Binary Cell Blockage Maps**

11
Step 1: Modeling Approach (cont’d)

- **Applying 2D shaping filter on 2D random signals**
  - Generated cell blockage maps should preserve given re-sampled forecast probabilities
  - However, a direct mapping from the 2D random signal lacks:
    - Spatial correlation between cells, and
    - Temporal evolvement between successive cell blockage maps

- **2D smoothing**
  - Model spatial correlation between cells

- **Cellular Automata**
  - Model evolvement (correlation) between successive maps

- **Model parameters obtained from historical weather data**
Step 1: Smoothing

- **Gaussian Kernel**
  - 1D: \( G_{1D}(x; \sigma) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{x^2}{2\sigma^2}} \)
  - 2D smoothing can be done by first smooth in the x direction, and then smooth the x-smoothed data, in the y direction

- **Full Width at Half Maximum (FWHM)**
  - Represents strength of spatial correlation
  - FWHM is related to standard deviation of the Gaussian kernel by
    \[
    \text{FWHM} = \sigma \sqrt{8 \ln 2}
    \]
Step 1: Smoothing (cont’d)

- Sample cell blockage map from direct mapping
Step 1: Smoothing (cont’d)

- Sample cell blockage map with adaptive smoothing
Step 1: Cellular Automata

Model details

- The independent blockage map $b'_{ij}$ is modified by $r_{ij}$ to yield a blockage map for the current time interval

$$b'_{ij} \rightarrow r_{ij} \geq r_0 \rightarrow N \quad r_{ij} \geq r_1 \rightarrow Y$$

$$0.5 < r_0 < 1, \quad r_0 + r_1 = 1$$

$$b_{ij} = [r_{ij} \geq (1 - b'_{ij})r_0 + b'_{ij}r_1]$$

- The value of $r_0$ reflects the influence of previous neighbor cells on current cell blockage - a large $r_0$ means weak influence
Step 1: Simulation Software

- Simulation software block diagram

Input

- Forecast Probabilities
- Re-sampling
- Probability Matrices

Pre-processing

- Random Matrix Generator
- Smoothing
- Mapping
- Intermediate Blockage

Simulation Loop

- Adjusting by Cellular Automaton
- Final Blockage

Output

- Initializing
- Previous Blockage
- Blockage Map Sequence
Step 1: Simulation Software (cont’d)

- Simulation software user interface
Step 1: Weather Scenario Animation
Step 2:
Develop efficient (fuel-optimal) conflict-resolution algorithm
Step 2: MIP for Conflict Resolution

Change heading and/or speed of each Aircraft
Step 2: MIP for Conflict Resolution (cont’d)

- Initial positions of the aircraft
- Determine Safety Regions using angles
  \[ \angle w_{i,j} \leq r_{i,j} \]
  \[ \angle w_{i,j} \geq l_{i,j} \]
Step 2: MIP for Conflict Resolution (cont’d)

\( \vec{v}_{0,i} \): Initial velocity of aircraft \( i \)
\( \vec{d}v_i \): Change in velocity of aircraft \( i \)
\( \vec{v}_{+,i} \): Velocity after change of aircraft \( i \)

Expand Constraint Equations with new variables

\[
\tan(w_{i,j}) = \frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} = \frac{v_{+,i,y} - v_{+,j,y}}{v_{+,i,x} - v_{+,j,x}} = \frac{v_{0,i,y} + d\vec{v}_{i,y} - v_{0,j,y} - d\vec{v}_{j,y}}{v_{0,i,x} + d\vec{v}_{i,x} - v_{0,j,x} - d\vec{v}_{j,x}} \geq \tan(l_{i,j})
\]

\[
\tan(w_{i,j}) = \frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} = \frac{v_{+,i,y} - v_{+,j,y}}{v_{+,i,x} - v_{+,j,x}} = \frac{v_{0,i,y} + d\vec{v}_{i,y} - v_{0,j,y} - d\vec{v}_{j,y}}{v_{0,i,x} + d\vec{v}_{i,x} - v_{0,j,x} - d\vec{v}_{j,x}} \leq \tan(r_{i,j})
\]
Step 2: MIP for Conflict Resolution (cont’d)

\[ f_0 = \sum_{i=0}^{n} g_{1,i} (\| \vec{v}_i \|) + \sum_{i=0}^{n} g_{2,i}(\theta_i) \]

Fuel cost for each plane  
Cost for changing direction

Cost function not convex in decision variables: \( \vec{d}v_i \)
Step 2: MIP for Conflict Resolution (cont’d)

Break up domain into regions

Apply linearization in each region using “basis”

\[
\begin{align*}
v_x &= c_0 \times x_0 + c_1 \times x_1 + c_2 \times x_2 \\
v_y &= c_0 \times y_0 + c_1 \times y_1 + c_2 \times y_2 \\
1 &= c_0 + c_1 + c_2 \\
v_{approx} &= c_0 \times v_0 + c_1 \times v_1 + c_2 \times v_2
\end{align*}
\]
Step 2: MIP for Conflict Resolution (cont’d)

Max Error < 1%
Step 2: Sample Solution
Step 2: Conflict Resolution Animation
Step 3:
Derive “probabilistic capacity” over time using Monte Carlo simulation that combines elements of Steps 1 and 2
Step 3: Definition of Capacity

- The **deterministic capacity “D(t)”** of a volume of airspace at time t (for a given traffic pattern and a given weather scenario) is the maximum number of aircraft that can enter that volume of airspace at time t and transit without having to be diverted to another volume of airspace or to an airport.
  - Not the maximum number of aircraft that can instantaneously exist within that volume of airspace.
Step 3: Definition of Capacity (cont’d)

- The stochastic capacity “$S(t)$” of a volume of airspace at time $t$ (for a given traffic pattern) is the probability distribution that is derived by solving for the deterministic capacity over the ensemble of weather scenarios that are deemed to be “consistent” with a given probabilistic weather forecast.
Step 3: Research Approach

- Conduct Monte Carlo simulations to determine stochastic capacity for a given traffic pattern and a given probabilistic weather forecast

  - For each weather scenario (from step 1)
    - Increase traffic volume until rerouting of traffic becomes infeasible
    - Traffic level at infeasibility is the “observation” of airspace volume for the given weather scenario

  - Set of observations over all weather scenarios is the cumulative probability distribution for given probabilistic weather forecast
Step 3: Results

- Simulation results converge to create an empirical cdf of capacity
Step 3: Results

\[ P(\text{cap} \cdot \ 10) = 0.01 \]
\[ P(10 < \text{cap} \cdot \ 20) = 0.03 \]
\[ P(20 < \text{cap} \cdot \ 30) = 0.07 \]
\[ P(30 < \text{cap} \cdot \ 40) = 0.54 \]
\[ P(40 < \text{cap} \cdot \ 50) = 0.25 \]
\[ P(50 < \text{cap} \cdot \ 60) = 0.08 \]
\[ P(60 < \text{cap}) = 0.02 \]

- \( P(\text{low cap}) = 0.04 \)
- \( P(\text{medium cap}) = 0.61 \)
- \( P(\text{high cap}) = 0.35 \)
Step 4:
Determine number of aircraft to send towards volume of airspace using probabilistic capacities and two-stage stochastic program
Step 4: Research Objective

- **Determine**
  - Departure times and cruise speeds for flights on the ground
  - Air holds and diversions for flights in the air

- **Such that**
  - Potential recourse actions based on realizations of capacity, such as air hold and diversion are considered
  - Number of flight in the sector does not exceed the sector capacity
  - Overall expected cost is minimized
Step 4: Research Objective (cont’d)

- **Inputs**
  - Probability distributions of airspace (i.e. sector) capacities over time, based on weather forecasts
  - Flight schedules
  - Cost structures (ground delay, speed change, air hold, diversion)

- **Output**
  - A dynamic algorithm that routes aircraft in response to evolving conditions in en-route sectors
Step 4: Research Approach

- **Implementation of the**
  - Original Stochastic Programming Model
  - Rolling Horizon Method in consideration of freezing flights

- **Computational results for two methods**
  - Solution Comparison
Step 4: Assumptions

- **Discrete time intervals**
  - Based on weather forecast resolution and capacity calculations
  - Typical implementation 15 minutes or more

- **Discrete probability distributions of capacities, generating scenarios**
  - Weather in each time interval: Good or Bad

- **First stage decisions**
  - Flights to depart in the current period and their cruise speeds
  - Flights to increase/decrease speed

- **Second stage decisions for each scenario**
  - Flights to accept in the sector
  - Flights to hold in the air
  - Flights to divert
Step 4: Stochastic Programming Model

\[
\begin{align*}
\min \quad & \sum_{m \in M} \left\{ \sum_{s=b_m}^{b_m+\Delta s_m} \sum_{t=s+\Delta k_m-\Delta t_m^+}^{s+\Delta k_m+\Delta t_m^-} \left[ (g_m^s + c_m^{t-s-\Delta k_m}) \cdot x_{st}^m \right] + \sum_{t=b_m+\Delta k_m-\Delta t_m^+}^{b_m+\Delta s_m+\Delta k_m+\Delta t_m^-} E\left[ d_m^t \cdot Q_m^t(\xi) \right] + \sum_{u=t}^{t+\Delta h_m} \left( a_m^{u-t} \cdot E\left[ P_m^{u-t}(\xi) \right] \right) \right\} \\
\text{s.t.} \quad & \sum_{m \in M} \sum_{t} p_{mt}^{tu}(\xi) \leq C_{mt}(\xi) \quad \forall u \\
& q_m^t(\xi) + \sum_{u=t}^{t+\Delta h_m} p_{mt}^{tu}(\xi) = \sum_{s} x_{st}^m \quad \forall t, m \\
& \sum_{s=b_m}^{b_m+\Delta s_m} \sum_{t=s+\Delta k_m-\Delta t_m^+}^{s+\Delta k_m+\Delta t_m^-} x_{st}^m = 1 \quad \forall m \\
& x_{st}^m \quad \text{binary} \quad \forall s, t, m \\
& p_{mt}^{tu}(\xi), q_m^t(\xi) \quad \text{binary} \quad \forall t, u, m
\end{align*}
\]
Computational Study

- Two levels of capacity (Good/Bad)
- Discrete time interval of 15 minutes
- Flight schedule for June 1, 2007 (TranStats data)
- Delta & AA flights with destination ATL
Rolling Horizon Method Parameters

- Number of periods considered (flights included): 10
- Number of future periods considered (flights not included): 2
- Freeze flights for following 2 periods
- Update every 1 period
- Include the latest result for repeated flights
- Remove flights that are diverted but could be reconsidered in future periods
## Comparison (Number of Period Considered = 10)

<table>
<thead>
<tr>
<th>flights depart in</th>
<th># of flight</th>
<th>Solve to optimality</th>
<th>Rolling horizon method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Objective value</td>
<td>Time (sec)</td>
</tr>
<tr>
<td>period 1-2</td>
<td>27</td>
<td>2,644.95</td>
<td>11.12</td>
</tr>
<tr>
<td>period 1-3</td>
<td>28</td>
<td>2,647.95</td>
<td>371.96</td>
</tr>
<tr>
<td>period 1-4</td>
<td>31</td>
<td>3,403.17</td>
<td>422.59</td>
</tr>
<tr>
<td>period 1-5</td>
<td>35</td>
<td>3,417.01</td>
<td>1,247.56</td>
</tr>
<tr>
<td>period 1-6</td>
<td>41</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>period 1-7</td>
<td>48</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>
Conclusions

- Developed a TFM methodology that will increase capacity and utilization

- Developed a mapping from weather forecast to airspace capacity

- Optimal or near-optimal framework for TFM in the presence of uncertainty
Thank You
Additional Slides
Step 4: Formulation

- **Input data including bounds on decision variables**

  - $M$: set of flights, $m$ as the index
  - $S$: set of time periods; $s$, $t$, $u$ as the index
  - $b_m^s$: scheduled departure period of flight $m$
  - $\Delta k_m$: regular flying period of flight $m$ to the sector
  - $\Delta s_m$: maximum number of periods flight $m$ can be ground-delayed
  - $\Delta t_m^+$: maximum number of periods flight $m$ can be scheduled to arrive early
  - $\Delta t_m^-$: maximum number of periods flight $m$ can be scheduled to arrive late
  - $\Delta h_m$: maximum number of periods flight $m$ can be air-held
Step 4: Formulation (cont’d)

- **Costs and capacity**
  - $g^s_m$: ground-delay cost for flight $m$ if it is sent at time $s$
  - $c^{t-s-\Delta k_m}_m$: speed-change cost for flight $m$ if it is sent at time $s$ and arrived the sector at time $t$
  - $a^{u-t}_m$: air-hold cost for flight $m$ if it arrives the sector at time $t$ and enters the sector at time $u$
  - $d^t_m$: diversion cost for flight $m$ which diverts at time $t$
  - $C^{u}_m$: sector capacity at time $u$

- **Variables**
  - $x^{st}_{m}$: 1 if flight $m$ is sent at time $s$ and arrives the sector at time $t$; 0 otherwise
  - $p^{tu}_{m}$: 1 if flight $m$ arrives the sector at time $t$ and enters the sector at time $u$; 0 otherwise
  - $q^t_{m}$: 1 if flight $m$ arrives the sector and diverts at time $t$; 0 otherwise
Step 4: SP Model for Rolling Horizon Method

\[
\begin{align*}
\min & \sum_{m \in M} \left\{ \sum_{s = b_m}^{b_m + \Delta s_m} \sum_{t = s + \Delta k_m - \Delta r_m}^{s + \Delta k_m} \left[ (g_m^s + c_m^{t-s-\Delta k_m}) \cdot x_{m}^{st} \right] + \sum_{t = b_m + \Delta k_m - \Delta r_m}^{t + \Delta h_m} \left[ E[d_m^t \cdot Q_m(\xi)] + \sum_{u = t}^{t + \Delta h_m} (a_m^{u-t} \cdot E[P_m^u(\xi)]) \right] \right\} \\
\text{st.} & \sum_{m \in M} \sum_{t} p_m^u(\xi) \leq C^u - \sum_{m \in M} \sum_{t'} p_{m}^{t'}(\xi) \quad \forall u \\
& d_m^t(\xi) + \sum_{u = t}^{t + \Delta h_m} p_m^u(\xi) = \sum_{s} x_{m}^{st} \quad \forall t, m \\
& \sum_{s = b_m}^{b_m + \Delta s_m} \sum_{t = s + \Delta k_m - \Delta r_m}^{s + \Delta k_m} x_{m}^{st} = 1 \quad \forall m \\
& x_{m}^{st} \quad \text{binary} \quad \forall s, t, m \\
& p_{m}^{u}(\xi), d_{m}^{t}(\xi) \quad \text{binary} \quad \forall t, u, m
\end{align*}
\]
Cost Parameters

- **Ground-Delay Cost**: at-gate & taxi with network effect by *Evaluating the true cost to airlines of one-minute of airborne or ground delay*

- **Speed-Change Cost**: approximation using Ren’s fuel-burn spreadsheet

- **Air-Holding Cost**: reference same as ground-delay cost

- **Diversion Cost**: ~ USD 40,000