Distributed Trajectory Flexibility Preservation for Traffic Complexity Mitigation

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Briefing Agenda

• Research Objectives and Questions

• Concept Description

• Trajectory Flexibility Metrics Definition and Estimation

• Trajectory Planning Algorithm and Cost Function

• Traffic Complexity Experiments and Preliminary Results

• Next Steps
Research Objectives and Questions

• Management of traffic complexity using trajectory-oriented as opposed to airspace-oriented perspective and approach
Research Objectives and Questions

Operational ATM Objectives
- e.g. Ensure Safety
- e.g. Ensure Stability
- e.g. Ensure Cost-effectiveness

Trajectory Constraints
- e.g. Separation Requirements
- e.g. Required Time of Arrival (RTA)

Trajectory Constraint Minimization
- Prevent Excessively Constraining Trajectory without Jeopardizing ATM Objectives

Trajectory Flexibility Preservation
- Preserve Ability to Manage Risk Exposure

Hypothesized Relationship

Traffic Complexity Prevention and Mitigation

Focus is on flexibility preservation and its impact on traffic complexity

• What is impact of trajectory constraint minimization on trajectory ‘flexibility’ preservation?

• What is impact of trajectory ‘flexibility’ preservation on traffic ‘complexity’ prevention and mitigation?
Research Objectives and Questions

• Research insight applicable to both centralized and distributed control issues
  
  – *Insight into appropriate allocation of air and ground functions, e.g.*,
  
  • Trajectory Flexibility Preservation conceived as air-based function
  
  • Constraint Minimization conceived as mostly ground-based function
  
  – *What is ‘Traffic Complexity’ in distributed, automated environment?*
  
  • Need suitable traffic complexity metric

• Develop concepts, metrics, methods, algorithms, and experiments, to investigate hypothesized relationships
Within conflict resolution look-ahead: e.g. avoid “coincidence” conflict

Conflict Resolution without Flexibility Preservation

- Previous conflict
- Look-ahead horizon
- Non-coordinated coincidental resolutions ready for execution
- Active routes
- Ownership
- Unrelated conflict pairs

Conflict Resolution with Flexibility Preservation

- Previous conflict
- Implicitly coordinated resolutions minimizing risk exposure to traffic
- Avoided “coincidence” conflict

Concept: Flexibility Preservation
Outside conflict resolution look-ahead: e.g. avoid congestion

Applicability of Trajectory Flexibility Prediction

- **Airborne flexibility function will question:**
  - Do I have enough flexibility to safely proceed?
  - Can I modify my trajectory to increase my flexibility?
  - Do I need to avoid this airspace entirely and replan?

Trajectories Designed to Preserve Flexibility

- **Hypothesis:**
  - If all aircraft apply flexibility preservation function, complexity automatically will be reduced.
Concept: Constraint Minimization

e.g., extending RTA tolerance

Solution Space before Constraint Relaxation

- Aircraft D
- Weather system
- Conflict free trajectories meeting RTA for aircraft A
- Conflict resolution look-ahead horizon
- Flexibility planning horizon
- Ownship aircraft A

RTA tolerance
- RTA at fix
- ETA at fix
- ETA range for conflict free trajectories meeting RTA

Solution Space after Constraint Relaxation

- Aircraft D
- Weather system
- Conflict free trajectories meeting RTA for aircraft A
- Conflict resolution look-ahead horizon
- Ownship aircraft A

Extended RTA tolerance
- RTA at fix
- ETA at fix
- Extended ETA range for conflict free trajectories meeting RTA

More flexible conflict free trajectories meeting RTA tolerance – Reducing aircraft A contribution to complexity

Aircraft B

Extended RTA tolerance
- RTA at fix
- ETA at fix
- Extended ETA range for conflict free trajectories meeting RTA

More flexible conflict free trajectories meeting RTA tolerance – Reducing aircraft A contribution to complexity

Aircraft B
Metrics Definition and Estimation

• Challenge: Lack of accepted metrics in literature for trajectory flexibility

• Starting point: Trajectory flexibility defined as its ‘ability to mitigate exposure to risk’
  - Defined more specifically as ability of trajectory to accommodate disturbances while meeting constraints (such as safety constraints or flow management constraints)

• Relevant trajectory characteristics
  - Robustness: Ability to remain feasible given disturbance
  - Adaptability: Ability to regain feasibility if feasibility is lost due to disturbance

• Corresponding metrics defined next
Approach: Start by analyzing trajectory solution space given limited degrees of freedom and in simple constraint scenarios

- (1) Varying only speed as degree of freedom
- (2) Varying only heading as degree of freedom
- (3) Varying both heading and speed as degrees of freedom
- Single RTA $\rightarrow$ RTA and conflict $\rightarrow$ Multiple RTAs and conflicts
Metrics Definition and Estimation

- Trajectory solution space for
  - *Single RTA at fix*
  - *Vary heading between $h_{\text{min}}$ and $h_{\text{max}}$*
  - *Fixed speed $V \rightarrow D = RTA \times V = \text{Constant}*$
Metrics Definition and Estimation

• Trajectory solution space for
  - *Single RTA at fix*
  - *Vary heading between* $h_{\text{min}}$ *and* $h_{\text{max}}$
  - *Vary speed between* $V_{\text{min}}$ *and* $V_{\text{max}}$
Metrics Definition and Estimation

- Discretization of trajectory solution space
  - Trajectory consists of discrete constant-speed, constant-heading segments
  - Solution space consists of series of conical shells each with heading range between $h_{\text{min}}$ and $h_{\text{max}}$
Metrics Definition and Estimation

• Discretization of trajectory solution space
  – Trajectory consists of discrete constant-speed, constant-heading segments
  – Solution space consists of series of conical shells each with heading range between $h_{min}$ and $h_{max}$ and speed range between $V_{min}$ and $V_{max}$
Metrics Definition and Estimation

- Discretization of trajectory solution space
  - Trajectory consists of discrete constant-speed, constant-heading segments
  - Solution space reduced by RTA and separation constraints
Metrics Definition and Estimation

- Flexibility definition extends to heading and speed: ability of trajectory to accommodate disturbances while meeting constraints
  - Constraint disturbances: introduction or modification of constraints
  - State disturbances: deviation of state from predicted trajectory
Metrics Definition and Estimation

• Trajectory flexibility metrics:

  – **Robustness**: *Ability of trajectory to remain feasible despite disturbance*

    • Metric: Probability of trajectory feasibility given distributions \( P_f(\text{traj}) \)
    
    • If state disturbance modeled by \( K \) possible trajectory instances and constraint disturbance modeled by \( C \) possible constraint instances, then
    
    \[
    \text{RBT(\text{traj})} = P_f(\text{traj}) = \sum_{i=1}^{K} P_i \times P_f(\text{traj}_i) \quad \text{where} \quad P_f(\text{traj}_i) = \sum_{c \in \text{Situations}} P_c \quad \text{where traj}_i \text{ is feasible}
    \]
    
    • If \( K \) trajectory instances are equally likely, RBT estimated by ratio:
    
    \[
    \text{RBT} (t, x, y) = \sum_{c=1}^{C} P_c \times \frac{f_c(t, x, y)}{f_c(t, x, y) + i_c(t, x, y)} \quad \text{where} \quad f_c(t, x, y) \text{ is number of feasible trajectories and } i_c(t, x, y) \text{ is number of infeasible trajectories, from (t, x, y) to destination, in situation c}
    \]

  – **Adaptability**: *Ability of trajectory to regain feasibility if lost due to disturbance*

    • Metric: \( \text{ADP} (t, x, y) = f(t, x, y) = \sum_{c=1}^{C} (P_c \times f_c(t, x, y)) \)
Metrics Definition and Estimation

Grid of discrete x-y cells

Reachability of point k over $\varepsilon$ given by $g_k$

Blocked cells due to loss of separation

Time step $\varepsilon$

Initialization of last time step:
$f=1$ if inside RTA tolerance, $f=0$ if outside

Point or cell k

Grid of cells $x,y$ over time $t$

Blocked cells due to loss of separation

Reachability of point k over $\varepsilon$ given by $g_k$

$f_c(t_{j-1},x,y)$ is derived by convolution of $g_k(x,y)$ and $f_c(t_j,x,y)$, sliding k in plane:

$$f_c(t_{j-1},x,y) = \sum \sum f_c(t_j,\sigma,\tau)g(x-\sigma,y-\tau)$$

or

$$f_c(t_{j-1},x,y) = 0 \quad \text{if } (t_{j-1},x,y) \text{ is not feasible}$$

Starting from final time step
Metrics Definition and Estimation

- Filtering for conflicts: Given: Intruder speed $V_{\text{int}}$ and heading $h_{\text{int}}$; ownship speed limits $V_{\text{min}}$, $V_{\text{max}}$ and ownship heading limits $h_{\text{min}}$ and $h_{\text{max}}$; and minimum separation $R$

- (1) Find four relative angles where $i$ and $j$ are set to “min” or “max”

$$h_{\text{rel},i,j} = \arctan\left(\frac{V_i \sin(h_j) - V_{\text{int}} \sin(h_{\text{int}})}{V_i \cos(h_j) - V_{\text{int}}(h_{\text{int}})}\right)$$

- (2) Find eight tangency points, $k = 1-8$

$$x_{\text{tan},k} = \pm \frac{R \times \tan(h_{\text{rel},i,j})}{\sqrt{\tan^2(h_{\text{rel},i,j}) + 1}} + x_{\text{int}}$$
$$y_{\text{tan},k} = \pm \frac{R}{\sqrt{\tan^2(h_{\text{rel},i,j}) + 1}} + y_{\text{int}}$$

- (3) Find 8 planes with norms $n_k$ and distances $d_k$:

$$n_k = \pm(\cos(h_{\text{rel},i,j}), \sin(h_{\text{rel},i,j}),0) \times \text{Unit}(V_i \cos(h_{\text{int}}), V_i \sin(h_{\text{int}}),1)$$

$$d_k = n_k \cdot (x_{\text{tan},k}, y_{\text{tan},k},0)$$

- (4) Center of cell $(t,x,y)$ is in conflict if and only if

$$(t,x,y) \cdot n_k < d_k, \forall k$$
Dynamic programming algorithm selected because

- Suitable to decision-tree formulation of the solution space and flexibility map
- Computational and storage load not an issue for non-real-time application

Three main steps:

1. Build the tree of state cells according to reachability
2. Starting from the last time step, use backward propagation over time to compute and store the optimal cost and tree path from each cell to the destination
3. Starting from the initial state, use forward propagation over time to connect the best cells in each time step to the destination
Trajectory Planning Algorithm

- Dynamic program back-propagation (recursion)

\[
Q(t, x(k), y(k)) = \min_{x, y : g_k(t+1, x, y) = 1} \{Q(t+1, x, y) + q(k \rightarrow (t+1, x, y))\}
\]

(1) Initialize cost at final time step
(2) Find and store best cost \( Q \) over reachable states
(3) Repeat (2) for each cell

Reachability of point \( k \) over time step \( \epsilon \) given by \( g_k \)

Grid of discrete \( x-y \) cells
Point or cell \( k \)
Trajectory Planning Algorithm

- Dynamic program Forward loop to build trajectory

(1) Start from initial cell

(2) Trace optimal path already stored, breaking ties randomly

Reachability of point k over $\varepsilon$ given by $g_k$
Trajectory Planning Algorithm

• Cost function

\[ Q(t,x(k),y(k)) = \min_{x,y: g_k(t+1,x,y)=1} \{Q(t+1,x,y) + q(k \rightarrow (t+1,x,y)) \} \]

• Four functions used for \( q \):
  - **Shortest path**
    \[ q(k \rightarrow (t+1,x,y)) = \text{distance}(k \rightarrow (t+1,x,y)) = \text{dist} \]
  - **Maximize adaptability**
    \[ q(k \rightarrow (t+1,x,y)) = -\text{ADP}(k) \]
  - **Maximize robustness**
    \[ q(k \rightarrow (t+1,x,y)) = -\text{RBT}(k) \]
  - **Tradeoff adaptability, robustness and path length \((T = \text{final time step})**
    \[ q(k \rightarrow (t+1,x,y)) = -\text{ADP}(k) - a^{T-t} \text{RBT}(k) + b^{T-t} \text{dist} \]
Traffic Complexity Analysis: Metrics

- (1) Lyapunov exponent representing predictability and organization
  (Delahaye, Peuchmorel, – ATM R&D 2003)

  Results from nonlinear dynamic system modeling of aircraft trajectories

  \[ \ddot{X}(t) = f(\dot{X}, t) \]

  \[ \min \sum_{i=1}^{N} \sum_{k=1}^{K} \left\| V_i(t_k) - f(\dot{X}_i, t_k) \right\| \]

  N is number of aircraft and K is number of samples per aircraft trajectory

  Using splines to ensure smoothness, a vector field is designed to minimize

  \[ \min \int_{R} \int_{R} \alpha \left\| \nabla \text{div}(\dot{X}, t) \right\|^2 + \beta \left\| \nabla \text{curl}(\dot{X}, t) \right\|^2 + \gamma \left\| \frac{\partial f(\dot{X}, t)}{\partial t} \right\| \, d\dot{X} dt \]

  \[ \alpha, \beta, \gamma \text{ positive real numbers controlling the smoothness of the approximation by focusing on constant divergence or constant curl} \]

- Lyapunov exponents representing sensitivity to initial conditions

  \[ \kappa(f) = \frac{1}{M} \sum_{m=1}^{M} \left\| D_x f(\phi(t_m, \dot{X})) \right\|_2 \]

  \[ D_x \text{ is the gradient matrix of the field at point } X \]

  \[ \phi \text{ is the point trajectory of the dynamic system at point } X \]
Traffic Complexity Analysis: Metrics

• (1) Lyapunov exponent representing predictability and organization

(Delahaye, Peuchmorel, – ATM R&D 2003)

– *High Lyapunov exponents represent high sensitivity to initial conditions*
  
  • Represent highly unpredictable future
  
  • Represent low organization of traffic (underlying dynamic system)
  
  • Represent high rate of change of relative distances

– *Low Lyapunov exponents represent low sensitivity to initial conditions*
  
  • Represent highly predictable future
  
  • Represent highly organized traffic
  
  • Represent low rate of change of relative distances
Traffic Complexity Analysis: Metrics

• (2) Consistency of flow pattern
  – Percentage of flights with the same flow pattern (scenario dependent)

• (3) Traffic proximity
  – Number of aircraft-minute with distance below a threshold

• Additional complexity metrics to be analyzed in future
  – For example, elements of dynamic density
  – For example, difficulty of conflict resolution
Traffic Complexity Analysis: Scenario

- Scenarios (1): Line of weather with two holes
  - Opposing traffic flows compete for two holes
    - One flow starts at (0, -120) ends at (0, 80)
    - Second flow starts at (0, 120) ends at (0, -80)
  - Symmetry ensures path length is not differentiator between the two holes
Scenarios (2): Round about around a hazard

- *Four intersecting flows go around the hazard*
  - Flow 1 starts at (0, -120) ends at (0, 80)
  - Flow 2 starts at (0, 120) ends at (0, -80)
  - Flow 3 starts at (-120, 0) ends at (80, 0)
  - Flow 4 starts at (120, 0) ends at (-80, 0)

- *Symmetry ensures path length is not differentiator between going around the hazard clockwise or counterclockwise*
Traffic Complexity Analysis: Scenario

• For both scenarios

  – Speed range 240-360 knots with 10-knot increments
  – Heading range +/- 60 degrees with 10-degree increments
  – Each aircraft meets an RTA (with no tolerance) at destination requiring path stretch
  – Each aircraft avoids hazards and preceding aircraft
  – Each aircraft generates trajectory once upon entry (no dynamic planning)
  – Each aircraft has one trajectory instance (no stochastic modeling) known exactly to the other aircraft
  – Separation zone set to 10 nautical miles to account for high uncertainty
  – Time increment 2 minutes
  – Grid cells of 2x2 nautical miles
Traffic Complexity Analysis: Scenario

• For both scenarios
  
  – **Compare traffic complexity metrics for five scenarios**

    • (a) Baseline: Minimize path length avoiding the hazards but not avoiding the traffic
      
      \[ q(k \rightarrow (t + 1, x, y)) = \text{distance}(k \rightarrow (t + 1, x, y)) = \text{dist} \]

    • (b) Minimize path length avoiding the hazards and traffic
      
      \[ q(k \rightarrow (t + 1, x, y)) = \text{distance}(k \rightarrow (t + 1, x, y)) = \text{dist} \]

    • (c) Maximize adaptability
      
      \[ q(k \rightarrow (t + 1, x, y)) = -\text{ADP}(k) \]

    • (d) Maximize robustness
      
      \[ q(k \rightarrow (t + 1, x, y)) = -\text{RBT}(k) \]

    • (e) Tradeoff adaptability, robustness and path length with \( a = 5000 \) and \( b = 40 \)
      
      \[ q(k \rightarrow (t + 1, x, y)) = -\text{ADP}(k) - a^{T-t}\text{RBT}(k) + b^{T-t}\text{dist} \]
Traffic Complexity Analysis: Scenario

• Example flexibility maps
Traffic Complexity Analysis: Results

- **Scenario 1, case a:** Shortest path length without traffic avoidance

Traffic pattern consistency:

Random selection of hole

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario1a_shortnoavoid.avi  
scenario1a_shortnoavoid.wmv
Traffic Complexity Analysis: Results

- Scenario 1, case b: Shortest path length with traffic avoidance

Traffic pattern consistency:

Unidirectional flow thru holes

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario1b_short.avi

scenario1b_short.wmv
Traffic Complexity Analysis: Results

- Scenario 1, case c: Maximum adaptability ADP

Traffic pattern consistency: Outer segment before hole, centerline after hole

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario1c_adapt.avi  
scenario1c_adapt.wmv
Traffic Complexity Analysis: Results

- **Scenario 1, case d**: Maximum robustness RBT (with respect to traffic)

Traffic pattern consistency:

Spread out to extremities

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario1d_robst.avi

scenario1d_robst.wmv
Traffic Complexity Analysis: Results

- **Scenario 1, case e:** Using tradeoff between adaptability, robustness and path length

Traffic pattern consistency:

- **Lyapunov exponent map:**
  High is red ← yellow/green → Low is blue

- **Unidirectional through holes**

- **scenario1e_shortadaptrobst.avi**
- **scenario1e_shortadaptrobst.wmv**
Traffic Complexity Analysis: Results

- Scenario 2, case a: Shortest path length without traffic avoidance

Traffic pattern consistency:
60% counterclockwise (blue)

Lyapunov exponent map:
High is red ⇐ yellow/green ⇒ Low is blue
Traffic Complexity Analysis: Results

- **Scenario 2, case b**: Shortest path length with traffic avoidance

Traffic pattern consistency:

- 68% counterclockwise (blue)

Lyapunov exponent map:
- High is red ↔ yellow/green → Low is blue

scenario2b_short.avi  
scenario2b_short.wmv
Traffic Complexity Analysis: Results

- Scenario 2, case c: Maximum adaptability ADP

Traffic pattern consistency:
97% counterclockwise (blue)

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario2c_adapt.avi
scenario2c_adapt.wmv
Traffic Complexity Analysis: Results

- **Scenario 2, case d**: Maximum robustness RBT (with respect to traffic)

Traffic pattern consistency:

70% counterclockwise (blue)

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario2d_robst.avi

scenario2d_robst.wmv
Traffic Complexity Analysis: Results

• Scenario 2, case e: Using tradeoff between adaptability, robustness and path length

Traffic pattern consistency:

84% counterclockwise (blue)

Lyapunov exponent map:
High is red ← yellow/green → Low is blue

scenario2e_shortadaptrobst.avi

scenario2e_shortadaptrobst.wmv
Traffic Complexity Analysis: Results

• Observations on traffic pattern consistency
  – Maximizing adaptability and robustness induced consistent traffic pattern compared to minimizing path length, particularly relative to shortest path without traffic avoidance
  – The resulting pattern depended on the objective function
    • Adaptability tended to concentrate trajectories as close to the centerline as possible
    • Robustness tended to spread traffic away from each other
    • Shortest path was closer to adaptability but not as close to the centerline
  – Pattern depends on action of early aircraft and may switch after lulls in traffic
  – Consistency may have been limited by lack of dynamic planning
  – Scenarios may have exhibited implicit coordination due to lack of variation in objective function among aircraft
Traffic Complexity Analysis: Results

• Lyapunov exponents – scenario 1
  (a) Shortest path w/o traffic avoidance
  (b) Shortest path with traffic avoidance
  (c) Maximum adaptability
  (d) Maximum robustness
  (e) Adaptability, robustness and path
Traffic Complexity Analysis: Results

• Lyapunov exponents – scenario 2

(a) Shortest path w/o traffic avoidance

(b) Shortest path with traffic avoidance

(c) Maximum adaptability

(d) Maximum robustness

(e) Adaptability, robustness and path
Traffic Complexity Analysis: Results

- Lyapunov exponents – scenario 1
  - *Maximizing adaptability (case c) consistently least complex*
  - *No consistent difference between other cases*
Traffic Complexity Analysis: Results

• Lyapunov exponents – scenario 2
  – Maximizing adaptability (cases c & e) consistently least complex
  – No consistent difference in the other cases

![Graph showing Lyapunov exponents over time for different cases](image-url)
Traffic Complexity Analysis: Results

• Observations on Lyapunov exponent maps
  
  – **Lyapunov exponent captures well the locations of complexity in terms of lack of predictability and variation in relative distance**
  
  • High exponents around aircraft wandering inconsistently with the surrounding traffic
  
  • High exponents around aircraft making sudden change in speed or heading
  
  • Distribution in location of high and low exponent values was consistent with observations about the traffic pattern consistency
    
    – **Relatively lower values where the traffic pattern was more consistent**
  
  • Average exponents (over map) consistently lower when adaptability is maximized
Traffic Complexity Analysis: Results

- Traffic proximity – scenario 1
  - Shortest path without traffic avoidance (case a) had worst proximity
  - Adaptability and robustness increased proximity relative to shortest path
Traffic Complexity Analysis: Results

- Traffic proximity – scenario 2
  - *Shortest path without traffic avoidance (case a) had worst proximity*
Next Steps and Future Research

• Further investigation of the impact of trajectory flexibility preservation on traffic complexity
  – Analysis of additional complexity metrics such as dynamic density

• Validation of adaptability and robustness metrics in terms of risk exposure mitigation

• Extending methods to within conflict resolution horizon

• Extending methods and algorithms to altitude degree of freedom

• Addressing computational load for real-time application in human-in-loop simulation and for actual use in flight
  – Particularly, in support of incorporating methods in NASA’s Autonomous Operations Planner (AOP)

• Extending methods and analysis to constraint minimization
Publications

• ATIO 2007 “A Distributed Trajectory-Oriented Approach to Managing Traffic Complexity”

• DASC 2007 “Distributed Traffic Complexity Management by Preserving Trajectory Flexibility”

• GNC 2008 “Trajectory Planning by Preserving Flexibility: Metrics and Analysis”

• ATCQ ASAS special issue 2009 “Metrics for Traffic Complexity Management in Self-Separation Operations”

• ATM R&D 2009 “Trajectory Flexibility Preservation for the Mitigation of Traffic Complexity”

• NASA contractor report 2009