Regional GDP — Extending Ground Delay Programs to Regional Airport Systems

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Abstract — Following the authors’ previous research on real-time intermodalism, this study proposes a Regional Ground Delay Program (Regional GDP) concept into the Collaborative Decision Making (CDM) system when a hub airport located in a regional airport system encounters a severe airside capacity reduction. It suggests that air traffic flow managers evaluate not only the imbalance of traffic demand and terminal capacity at the hub airport but also excess capacity at other airports in the same region, assuming that airlines could incorporate ground modes into their disruption management and use ground vehicles to transport passengers and crew members between original scheduled and diverted airports. A mathematical programming model is established to help air traffic flow managers make decisions on initiating a Regional GDP advisory. A case study at San Francisco International Airport (SFO) was conducted. Results show that the initiation of a Regional GDP is suggested on a day with severe weather conditions.

Keywords—Ground Delay Program (GDP), Regional Airport Systems, Capacity, Delay, Congestion, Regional GDP

I. INTRODUCTION

Under the current Collaborative Decision Making system (CDM), as illustrated in Fig. 1 [1], once a severe imbalance of traffic demand and airport capacity supply is detected, a GDP advisory is issued. This advisory assigns scheduled flight Estimated Arrival Times (EATs) to that airport, most of which are later than the original Scheduled Arrival Times (SATs). Airlines respond to this advisory by canceling flights. If the imbalance is resolved by the airlines’ adjustments, the GDP is cancelled; otherwise, the air traffic control system command center (ATSCCC) issues each remaining scheduled flight an Expected Departure Clearance Time (EDCT) and a Controlled Time of Arrival (CTA). Airlines manage their CTAs (or arrival slots) in their best internal business interests. An airline operation center (AOC) may cancel more flights, re-order flight sequences, and re-assign flights to CTAs to utilize the arrival slots of delayed or cancelled flights. This flexibility of employing slots under CDM encourages airlines to cancel and re-order flights, thereby reducing air traffic demand, passenger delay, and disruption cost.

However, when the imbalance is severe, the function of a single airport GDP, as described above, is limited. This is because airlines respond to a GDP advisory by canceling less costly flights first. With the number of cancellations growing, the marginal cost of canceling a flight increases; in other words, the marginal benefit of GDP advisory decreases. Thus, for severe traffic-demand-and-airport-capacity imbalances, we propose a Regional GDP by extending a single airport GDP to a regional airport system so that excess capacities at other airports in the same region could be accessed and utilized for alleviating delays and congestion at the referenced airport. A regional airport system is defined as all commercial airports within 50 miles of a referenced airport, usually a large hub airport [2]. This definition can be adjusted depending not on the distance but on the driving time of ground modes from a referenced airport to other airports when the inter-modal idea is introduced, as elaborated in Section 2.

Various causes lead to airside capacity reductions at an airport, such as adverse weather, equipment outages, runway collisions, terrorism threats, and others. Since airports in a regional airport system are close to each other, it is likely that adverse

1 Airline Operation Centers (AOC) centrally manage the operations of airline resources, monitor the safety of operations, and exchange critical information with governmental authorities and other airlines.
weather would affect all the airports and thus diminish excess capacities that could be used to help the busy hub airport. However, some busy hub airports have parallel runways that are too close to each other and thus can be treated only as a single runway under Instrument Flight Rules (IFR) conditions once weather conditions are undesirable. Other airports in the same region may have simpler runway configurations that would not be affected by the adverse weather as much as the referenced airport.

Figure 1 Flow Chart of Existing Collaborative Decision Making

In addition, airport capacities at hub airports would be more dependent on equipment once the Next Generation Air Transportation System (NextGen) was established. NextGen is the Federal Aviation Administration’s (FAA) plan to modernize the National Airspace System (NAS) through 2025. As stated by FAA, the objective of NextGen is to “provide additional airport capacity, and thereby reduce delays, through enhanced air traffic control (ATC) techniques and technologies.”[3, page 2] That means airport capacities would be dependent on equipment facilitating those techniques and technologies. For instance, required navigation performance (RNP) increases the number of aircraft that can safely use a particular airspace and therefore accommodate the increasing demand for air traffic capacity; however, monitoring and alerting facilities are critical for realization of this new method. Another example is Very Closely Spaced Parallel Runway (VSCPR) operations. One of the critical requirements for these operations is Precision Runway Monitor (PRM), and a related equipment outage would conceivably affect airport capacity at a larger magnitude than current-day equipment outages. All these trends point to utilizing excess capacities at other airports in a regional airport system.

The next section illustrates CDM with Regional GDPs and introduces a real-time inter-modal idea into the implementation of the Regional GDP. Section 3 proposes a mathematical programming model to support the decision of initiating the Regional GDP. A case study is presented in Section 4, followed by conclusions in Section 5.

2 “Required Navigation Performance (RNP) is defined by ICAO as "a statement of the navigation performance necessary for operation within a defined airspace." Part of a broader concept called "Performance-based Navigation," RNP is a method of implementing routes and flight paths that differs from previous methods in that not only does it have an associated performance specification that an aircraft must meet before the path can be flown but it must also monitor the achieved performance and provide an alert in the event that this fails to meet the specification.” Wikipedia.com, accessed on Nov. 15, 2008.
II. CDM WITH REGIONAL GDP

A flow chart of CDM with Regional GDP is illustrated in Fig. 2. Similar to the existing CDM, air traffic flow managers first evaluate the imbalance of traffic demand and airport capacity at one airport, usually a large hub airport. If the airport is located within a regional airport system, instead of proposing a GDP advisory based on the single-airport evaluation, air traffic flow managers could assess (with inputs from airlines) whether a Regional GDP is needed. If it is not needed, CDM follows the existing procedure until the GDP has ended. Otherwise, air traffic flow managers propose a Regional GDP advisory that includes a GDP advisory at the hub airport and information regarding available slots at other airports in the regional airport system. Airlines respond to this Regional GDP advisory by canceling flights to the hub airport and diverting flights to nearby airports. The nearby airports with
diversions are termed “alternative hubs” of the original hub airport. Given airline diversion plans, air traffic flow managers assign slots at other airports according to the ration-by-schedule (RBS) algorithm and make sure the excess capacity levels at other airports would not be exceeded. Once slots at other airports are settled, airlines can respond by canceling more flights, re-ordering flights, and finalizing diversions. Air traffic flow managers then evaluate if the GDP at the original hub airport is still required. If the answer is negative, the GDP advisory at the original hub airport is cancelled; if the answer is positive, it follows the remaining steps in the existing CDM.

One important decision that air traffic flow managers have to make is whether a Regional GDP is needed or not. In the present implementation of CDM, FAA and the airlines provide and share information and are major players of the GDP. With a Regional GDP, a broader aviation community including airports and related ground transportation providers is involved. Calling a redundant Regional GDP will cause unnecessary cost. In the next section, we propose a mathematical program to help air traffic flow managers make the decision. Before starting Section 3, however, we would like to introduce the concept of real-time inter-modal diversion (RTIMD) into the Regional GDP.

RTIMD suggests setting up ground transportation connections between airports within a regional airport system if diversions of original-hub-bound flights to other airports are needed. In the current system, under some circumstances, such as severe airport capacity shortfalls or closures due to weather, equipment outages, or emergencies, flights have to be diverted to alternate airports instead of landing at their original destinations. Once a flight is diverted, it remains at the alternate airport until clearance is received from Air Traffic Control ensuring that its original destination has enough capacity to allow it to proceed there. Michael Irrgang conducted delay estimations of flight diversions [4] and estimated that the total delay caused by diversions ranges from 85 minutes to 2 hours plus the destination airport closure time. This total includes:

- 10 - 20 minutes, extra flying time to reach the alternate airport
- Original destination closure time (30-75 minutes refueling time at the alternate airport is also included in this period)
- 30 - 60 minutes, waiting for new departure clearance to original destination
- 45 minutes, flying to original destination

In the current system, the total demand at the destination airport is not reduced as the result of diversions because diverted flights must ultimately fly to it. Demand from diverted flights creates additional delays for flights that are not diverted, further compounding the problem. The situation worsens if the capacity at the destination airport is affected for a longer duration or the capacity shortfall or airport closure occurs in the early hours of the day. RTIMD, in contrast, avoids flying aircraft back to the original hub airport, thus avoiding associated costs and delays. By utilizing nearby airports in the same region and the ground transport connection between them, an airline can integrate its operations at the original hub and alternative airports. For more details on RTIMD, please refer to the first author’s dissertation “Real-time Inter-modal Strategies for Airline Schedule Perturbation Recovery and Airport Congestion Mitigation under Collaborative Decision Making (CDM)”[5].

III. DECISION SUPPORT FOR INITIATING A REGIONAL GDP

A mathematical programming model was developed to help air traffic flow managers make a decision on whether a Regional GDP is needed. Binary decision variables are equal to 1 if flights should be diverted to other airport, 0 otherwise. The objective function is total disruption cost. If it is profitable system-wide to divert flights to other airport(s), a Regional GDP should be initiated, otherwise not.

The inputs of the model would include the airlines’ original schedules, capacity profiles at the hub airport, information used in the existing CDM, and excess capacities at other airports in the regional airport system. To make the model work, airlines also need to provide additional information such as the numbers of passengers who purchased itineraries going through the original hub airport, the percentage of transfer passengers on the flights, and the ranking of importance of flights. A more important flight would be less likely to be diverted to other airports in disruption management decisions. The airline-specific information is confidential and should be accessible only to air traffic flow managers, and not to other players in the CDM.

A. Objective Function

The objective function of the mathematical model is a total disruption cost, including passenger disruption cost, airline disruption cost, and regional system cost if a Regional GDP were to be initiated. Passenger disruption cost includes passenger delay
cost, passenger extra transportation time if flights are diverted to alternative hubs, and transfer passengers’ misconnection cost. At this stage of the CDM, air traffic flow managers assume that airlines either let flights fly into the original hub airport or divert them to alternative hubs and leave cancellation decisions to the airlines later. Thus, airline disruption cost includes only the delay cost of flights to the original hub airport and the diversion cost to alternative hubs. The regional system cost includes miscellaneous costs of implementing a regional GDP such as administrative cost, ground transportation operating cost, common-use ground facility cost at alternative hubs, and others. Hence, the objective function can be expressed with equation (1). The set of decision variables in this objective function, \( x_{ij} \), equals 1 if Flight \( i \) is diverted to an alternative hub \( j \), 0 otherwise. Notations used for this Regional GDP Decision Support Model are listed in Appendix I.

\[
\text{Min} \left\{ \sum_k w_k \cdot P_k + \sum_{ij} x_{ij} \cdot BT_{ij} \cdot Pax_i \right. \\
\left. + \sum_{ij} x_{ij} \cdot TPax_j \cdot Mis_j \right\} \cdot C^P \\
+ \sum_k w_k \cdot F_k \cdot C^F + \sum_{ij} x_{ij} \cdot C^D_j + \sum_{ij} C^C_j y_j \right\} \\
\text{with:} \\
\begin{align*}
w_k &= \min \left\{ \text{max} \left( 0, \frac{D_k}{c_i} - t_k \right) \right. \\
&\left. \text{max} \left( 0, \frac{D_k - c_i T_j}{c_T} - (t_k - T_j) \right) \right. \\
&\left. 0 < t_k \leq T_j \right. \\
&\left. \text{max} \left( 0, \frac{D_k - c_i T_j}{c_T} - (t_k - T_j) \right) \right. \\
&\left. t_k > T_j \right. \\
D_k &= \sum_{i \in [\text{flights}]} \left( 1 - \frac{x_{ij}}{\text{Pax}_i} \right) \\
\forall k \in K \\
P_k &= \sum_{i \in [\text{flights}]} \left( 1 - \frac{x_{ij}}{\text{Pax}_i} \right) \cdot \text{Pax}_i \\
\forall k \in [1..K] \\
F_k &= \sum_{i \in [\text{flights}]} \left( 1 - \frac{x_{ij}}{\text{Pax}_i} \right) \\
\forall k \in [1..K] \\
\end{align*}
\]

The components of the objective function are described as follows.

The term \( \sum_k w_k \cdot P_k \) represents the delay of passengers on flights that land at the original hub airport, where \( w_k \) is the average delay of flights landing at the original hub airport during time period \( k \), and \( P_k \) is the total number of passengers on those flights, calculated using Equation (4). \( w_k \) is obtained by implementing the delay continuous approximation method described in Appendix II. That method suggests close forms that can be used in optimization programming models. A performance analysis with different time units demonstrated that the inaccuracy when using the approximation method decreases when the daily arrival rate increases. It becomes less than 10 seconds when the daily arrival rate reaches 150 arrivals per day.

The term \( \sum_i \sum_j x_{ij} \cdot BT_{ij} \cdot Pax_i \) represents extra ground transportation time for transfer passengers whose flights are diverted to alternative hubs, where \( x_{ij} \) is the decision variable, \( BT_{ij} \) is the ground transportation time for passengers on Flight \( i \) from the alternative hub \( j \) to the hub airport, and \( Pax_i \) is the number of passengers on Flight \( i \).

The term \( \sum_i \sum_j x_{ij} \cdot TPax_j \cdot Mis_j \) represents the estimated misconnection cost for transfer passengers. It depends on \( TPax_i \), the number of transfer passengers on Flight \( i \), and \( Mis_j \), the estimated unit penalty of missing connections.

The term \( \sum_j C^D_j \cdot y_j \) represents the cost of utilizing airports as alternative hubs, where \( C^D_j \) is a cost parameter and \( y_j \) is a decision variable indicating that airport \( j \) is used as an alternative hub.

The term \( \sum_k w_k \cdot F_k \cdot C^F \) represents the flight delay cost of flights that plan to land at the original hub airport, where \( w_k \) is the average delay of flights landing at the original hub airport during time period \( k \) and \( F_k \) is the total number of flights obtained from Equation (5).

\( \sum_i \sum_j x_{ij} \cdot C^D_j \) represents the flight diversion cost.
B. Parameters

The parameter $C^p$ is the passenger value of time. According to a report from GNA Inc., "Economics Values for FAA Investment and Regulatory Decisions, A Guide," the value of passenger time per hour for all air travel purposes is $28.6 [6].

$C^f$ is the parameter of the flight delay cost. A comprehensive report by the University of Westminster lists tactical ground delay costs at the gate and while taxiing with network effect [7]. The delay cost varies for different aircraft types and also is dependent on delay lengths. In our model, an aggregate flight delay cost is needed. Based on numbers from that report, we assume the flight delay cost per hour is $2,000.

$C_{ij}^D$ is the diversion cost for flight $i$ being diverted to an alternative hub $j$. It is difficult for each airline to provide flight-specific diversion costs, and it is also difficult to prevent airlines from gaming while submitting the diversion cost. One method to manage the difficulties is as follows: (1) determine rankings of flights only from airlines, assuming that the lower the ranking of the flight in an airline’s schedule, the less likely that it would be diverted to other airports in the airline’s individual disruption management decision; (2) create diversion cost ranges based on airline consensus; and (3) calculate flight-specific diversion costs according to the percentile of ranking of the flight in an airline’s schedule. In the case study presented in Section 4, a simpler version of this method is used to determine the diversion cost. There is no literature providing references for determining a diversion cost range. Arguello and Yu developed a table of flight cancellation costs in their study for aircraft routing in response to grounding and delays [8]. The cancellation cost is $3,555 to $6,585. For experimental purposes, we can assume an average diversion cost is about $5,000 per flight.

C. Constraints

The minimization of the objection function (1) is subject to the following constraints:

1. A flight can be diverted to alternative hub $j$ only where the runway length at airport $j$ satisfies the landing requirement

$$x_{ij} = 0 \quad \forall \Lambda_{ij} = 0$$  \hspace{1cm} (6)

where $\Lambda_{ij}$ indicates if the runway length requirement is satisfied, 0 otherwise.

2. A flight can be diverted to, at most, one alternative hub

$$\sum_j x_{ij} \leq 1 \quad \forall i \in I$$ \hspace{1cm} (7)

3. Flights can be diverted to alternative hub $j$ only if airport $j$ is used as an alternative hub

$$\sum_i x_{ij} \leq M \cdot y_j \quad \forall j \in \Gamma$$ \hspace{1cm} (8)

4. The total number of diverted flights to alternative hub $j$ cannot exceed the excess capacity at the alternative hub

$$\sum_{i \in \Gamma(n_i - \text{HSAn}_i < 0)} x_{ij} \leq ECap_{n_j} \quad \forall j \in \Gamma \forall n \in N$$ \hspace{1cm} (9)

The optimization will give out a shadow price of constraint 4 if it is bonded. The shadow price gives the reduction of objective function if one extra arrival slot at the alternative hub is released for the hour that excess capacity constraint is bounded. Given the operating situation at the alternative hub, air traffic flow managers may evaluate the impact of the slot release to the alternative hub operations.

IV. A CASE STUDY

San Francisco International Airport (SFO) was selected as the hub airport in our case study. SFO was located in San Francisco Regional Airport System, which also includes Oakland International Airport (OAK) and Mineta San José International Airport (SJC). Individual SFO-bound flight information on June 25, 2008, a day with adverse weather conditions, was obtained from the Aviation System Performance Metrics (ASPM) database maintained by FAA. Airlines operating less than seven flights that day were excluded from the data set, reducing the total scheduled arrivals at SFO from 540 to 429. The scheduled and actual arrival times of the 429 flights are shown in Fig. 3. The upper curve depicts cumulative arrivals according to their scheduled arrival times. Each scattered dot in the figure is one actual arrival time of one flight. The horizontal difference between the scheduled arrival curve and one scattered dot is the delay of one specific flight. The longest flight delay on that day was about five hours for an American Airlines (AA) flight originally scheduled to land at 7:58pm but which actually landed after 12:00 midnight. The bottom curve depicts cumulative arrivals according to their scheduled arrival times. Each scattered dot in the figure is one actual arrival time of one flight. The horizontal difference between the scheduled arrival curve and one scattered dot is the delay of one specific flight. The longest flight delay on that day was about five hours for an American Airlines (AA) flight originally scheduled to land at 7:58pm but which actually landed after 12:00 midnight. The bottom curve depicts cumulative arrivals according to ordered actual arrival times. The slopes of the curve represent realized arrival capacity rates, which are 27 arrivals per hour from 8:00am to 4:00pm and 22 arrivals per hour afterwards. These numbers are slightly lower...
than the hourly Arrival Acceptance Rates (AARs) recorded in ASPM (because the called rates are not always being fully utilized). We use that curve as the capacity profile at SFO for that day. For simplification, only OAK is used as the alternative hub candidate in this case study since it is relatively close to SFO. The excess hourly capacities at OAK are obtained by subtracting arrival demand from AARs that also were obtained from ASPM.

![Cumulative number of arrivals](image)

**Figure 3 Cumulative Numbers of Arrivals at SFO**

In this case study, airline-specific information is synthesized in the following way: (1) given equipment type and average load factor of flights to SFO, the number of passengers who purchased itineraries for SFO-bound flights is calculated by multiplying seat capacity by a load factor randomly generated around that average load factor; (2) the number of transfer passengers is calculated by multiplying passenger numbers by an average transfer passenger percentage; (3) as shown as the scattered dots in Fig. 3, there are two sets of flights: one is on the left side of the capacity profile curve (bottom curve) and the other is on the right side. If airlines have not reordered their flights, the scheduled flights would have landed at SFO following the sequence illustrated by the capacity profile curve. So, it is reasonable to say that the flights on the left side of the curve, with less delay than what was supposed to be, are more important than those on the right side. We assume the diversion cost of flights on the left side is $5,000, the average diversion cost defined in Section 3, and the diversion cost of flights on the left side is $7,500.

The MINLP solver on NEOS Server 4.0[^3] was used to perform the minimization of the mathematical programming model. The design, implementation, and more details of the NEOS Server were discussed by Cayak et al. [9], Gropp et al. [10], and Dolan [11].

The optimization of the mathematical programming model suggests 45 flights being diverted to OAK; thus, a Regional GDP is needed. If flights landed at SFO following the realized capacity profile curve without airlines’ reordering, the longest flight delay would be 2 hours. In comparison, the longest flight delay after diverting the 45 flights is 30 minutes. The model is designed to assist with the decision to initiate a Regional GDP. The flight delay

comparison would be quite different after airlines cancelled, re-ordered, and diverted flights according to their internal interests.

V. CONCLUSIONS

This study proposes a Regional GDP concept when a hub airport located in a regional airport system encounters a severe airside capacity reduction. It suggests that air traffic flow managers evaluate not only the imbalance of traffic demand and terminal capacity at the hub airport but also excess capacities at other airports in the same region. Enhanced CDM proposing this new concept is illustrated. A mathematical programming model was established to help air traffic flow managers make decisions on initiating a Regional GDP advisory. This study also suggests that airlines use ground vehicles to transport passengers and crew members between the hub airport and alternate airports without ferrying diverted aircraft back to the hub airport if it is more cost-effective. By doing this, the diverted aircraft can continue flying to its next segment. According to the definition of a regional airport system, the average travel time between two airports in the regional airport system would be about half hour, and the maximum should not exceed one hour. Thus, if the ground connection could be organized efficiently, the passengers and crew members moving between the two airports could be imagined as moving between different terminals within one mega-airport. In addition, the ground connection can be used to reallocate crew members, which are resource constraints in airline recovery. A case study at SFO was conducted. Results show that on a day with severe weather conditions, an initiation of the Regional GDP is suggested.

Ball et al. pointed various research directions for CDM in air traffic management [12]. This study goes beyond their report but was heavily inspired by their ideas in collaborative resource allocation methods. It also echoes the metroplex airspace management research that promoted by National Aeronautics and Space Association (NASA). We envision that diverting flights to other airports would alleviate airspace congestion in the vicinity of a busy and overwhelmed large hub airport, yet the impact would be system-specific. Further studies and simulations would be needed to quantitatively describe the impact. Cost parameters are critical inputs of the Regional GDP decision support model. The study conducted by the University of Westminster offered an industry-wide cost of delay [6]. Similar studies should be conducted for U.S. airlines so air traffic flow managers can have benchmark cost for their decision making.

REFERENCES


AUTHOR BIOGRAPHIES

Yu Zhang is an assistant professor in the Department of Civil and Environmental Engineering at the University of South Florida. Her research aspires to develop methods for next generation air transportation systems to ensure efficiency, robustness, safety, and sustainability notwithstanding increasing demands and external uncertainties. She received master’s and doctoral degrees from UC
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### APPENDIX I: NOTATION FOR REGIONAL GDP DECISION SUPPORT MODEL

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<thead>
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<th>Notation</th>
<th>Category</th>
<th>Description</th>
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<tr>
<td>$BT_j$</td>
<td>Input</td>
<td>Ground transportation time from the alternative hub $j$ to the original hub for flight $i$</td>
</tr>
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<td>$C_I$</td>
<td>Input</td>
<td>Reduced capacity rate at the original hub airport</td>
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<td>$C_V$</td>
<td>Input</td>
<td>Resumed capacity rate at the original hub airport</td>
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<td>$C_{j}^A$</td>
<td>Parameter</td>
<td>Cost of utilizing airport $j$ as an alternative hub</td>
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<tr>
<td>$C_{i,j}^D$</td>
<td>Parameter</td>
<td>Diversion cost for flight $i$ being diverted to an alternative hub $j$</td>
</tr>
<tr>
<td>$C_F$</td>
<td>Parameter</td>
<td>Parameter of flight delay cost</td>
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<tr>
<td>$C_P$</td>
<td>Parameter</td>
<td>Passenger value of time</td>
</tr>
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<td>$D_k$</td>
<td>Intermediate Variable</td>
<td>Cumulative number of arrivals through time period $k$</td>
</tr>
<tr>
<td>$\Lambda_{ij}$</td>
<td>Input</td>
<td>Indicator of runway qualification (equals 1 if the runway length requirement for flight $i$ is satisfied, 0 otherwise)</td>
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<td>$E\text{Cap}_{nj}$</td>
<td>Input</td>
<td>Hourly excess capacity at alternative hub $j$</td>
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<td>$F_{ik}$</td>
<td>Intermediate Variables</td>
<td>Total number of flights remains flying to SFO for time period $k$</td>
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<tr>
<td>$HSA_i$</td>
<td>Input</td>
<td>Indicator for which hourly time period</td>
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<th>Category</th>
<th>Description</th>
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<td>$i \in I$</td>
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<td>Arrival flights</td>
</tr>
<tr>
<td>$j \in J$</td>
<td>Index</td>
<td>Airports other than the original hub in the system</td>
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<td>$M$</td>
<td>Parameter</td>
<td>A very large number</td>
</tr>
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<td>Penalty for passengers on flight $i$ assume there is certain possibility that they may miss their connection</td>
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<td>Input</td>
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<td>Input</td>
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<tr>
<td>$w_k$</td>
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<tr>
<td>$y_j$</td>
<td>Decision Variable</td>
<td>Equals to 1 if airport $j$ is used as an alternative hub, 0 otherwise</td>
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### APPENDIX II: CONTINUOUS APPROXIMATION DELAY ESTIMATION METHOD

The average delay of flights that have landed at the original hub airport during time period $k$, $w_i$ (see Equation 2), is calculated using the continuous approximation method as shown in Fig. II-1. As can be seen, the cumulative number of scheduled arrival flights is approximated as a continuous curve on the left. The piecewise line on the right represents the cumulative number of arrivals restricted by a capacity shortfall at a hub airport. To obtain a closed form for flight delay, the time of day is divided into a finite set of time periods of equal duration. For flights whose scheduled time is in time period $k$, the flight delay is either:
\[
\begin{align*}
\max\left(0, \frac{D_k}{c_I} - t_i\right) \\
\min\left(\max\left(0, \frac{D_k}{c_I} - t_i\right), \max\left(0, \frac{D_k - c_I T_{I_k}}{c_I} - (t_i - T_{I_k})\right)\right)
\end{align*}
\]

\[
\max\left(0, \frac{D_k - c_I T_{I_k}}{c_I} - (t_i - T_{I_k})\right)
\]
depending on whether the time period \(k\) is before or after the time period when the capacity recovers. In these expressions, \(D_k\) is the cumulative number of arrival flights up to time period \(k\), \(c_I\) is the capacity level during disruption, \(c_I\) is the normal capacity level, and \(T_{I_k}\) is the length of capacity shortfall.

To test the performance of the continuous approximation method, mean absolute differences of average flight delay between an exact calculation method and a continuous approximation are compared. The experiment is constructed as follows: (1) a series of daily arrival rates is assumed, from 50 to 300 arrivals per day; (2) the sequence of arrivals in one day is created randomly, and 20 repetitions are used for each daily arrival rate; (3) the time of day is divided into two finite sets of time periods with equal durations of 2 minutes and 5 minutes. The results demonstrate, as shown in Fig. II-2, the mean absolute difference decreases when the daily arrival rate increases. It becomes less than 10 seconds when the daily arrival rate reaches 150 arrivals per day. Daily arrival rates at all large hub airports in the U.S. are more than 150; hence, calculated delays from the continuous approximation method are close to those from the exact solution. This method provides a closed form that can be used in the Regional GDP model described above.