Motivation

- Mass of aircraft at any time is a key driver of its behavior
  - Factor in determining fuel burn, and consequently emissions
  - Factor in determining performance, e.g., trajectory

- Initial mass (also known as Takeoff Weight, or TOW) is of particular interest, and is an essential input to most trajectory prediction, as well as fuel burn and emissions estimation tools

- TOW of a flight is considered proprietary, and is generally not disseminated or known
Related literature

- Estimation of TOW by combining component weights
  - Empty weight + fuel + payload [AEDT 2016, Sherry & Neyshabouri 2014]


- Linear regression models of TOW using airline-derived data, considering stage length, runway length, etc. [Clarke et al. 2016]

- Modeling of TOW using ADS-B data, with assumptions on takeoff thrust, coefficient of friction, etc. [Sun et al. 2016]

- Bayesian inference of TOW, combining phase-based models [Sun et al. 2017]

- Limited validation, due to lack of ground truth data
Problem statement

- Can the takeoff weight of a flight be inferred from trajectory data corresponding to its takeoff roll?

- What is the variability (and correspondingly, the uncertainty) associated with this TOW estimate?
Leveraging “physics” in feature selection

\[ L + N = mg \]
\[ F_n - D - f_r = ma \]
\[ L = qSC_L \]
\[ D = qSC_D \]
\[ f_r = \mu N \]
\[ a = \frac{dV}{dt} \]
\[ q = \frac{1}{2} \rho V^2 \]
\[ F_n = F_n(n_{\text{eng}}, V, \eta, \rho, F_{00}) \]
\[ R = \int_{V_1}^{V_2} V \frac{dV}{da} \]

\[ m = m(R, \rho, V_1, V_2, S, F_{00}, n_{\text{eng}}, C_L, C_D, \mu, \eta) \]

- **Observed**
- **Constant for given aircraft/engine**
- **Unknown**
Simplifying assumptions

- Ground speed used as surrogate for true airspeed (wind effects neglected)
- Aircraft mass assumed constant during takeoff roll
- Lift and drag coefficients assumed constant during takeoff roll
- Net thrust depends on the number of engines \( (n_{\text{eng}}) \), aircraft velocity \( (V) \), deration level \( (\eta) \), ambient air density \( (\rho) \), and the maximum sea level, static engine thrust \( (F_{00}) \)
## Selected features

<table>
<thead>
<tr>
<th>Input Variables ($x$)</th>
<th>Output variable ($y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Takeoff ground roll distance ($R$) in m</td>
<td>• TOW ($m$) in kg</td>
</tr>
<tr>
<td>• Ambient air density ($\rho$) in kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>• Aircraft velocity (ground speed) at start of takeoff ground roll ($V_1$) in m/s</td>
<td></td>
</tr>
<tr>
<td>• Aircraft velocity (ground speed) at end of takeoff ground roll ($V_2$) in m/s</td>
<td></td>
</tr>
</tbody>
</table>

- Unmodeled factors contribute to the uncertainty of TOW estimates, and are reflected in prediction intervals
Regression methods

- Continuous and metric variables: Regression problem

- Range of methods investigated:
  - Ordinary Least Squares (OLS) Regression
  - Classification and Regression Trees (CART)
  - Least Squares Boosting using Trees (LSB)
  - Hierarchical Bayesian Least Squares Regression
  - Gaussian Process Regression (GPR)
Gaussian Process Regression (GPR)

- Nonparametric, probabilistic method

\[ y_i = f(x_i) + \epsilon_i, \epsilon_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2_n) \]

\[ f \sim \mathcal{GP}(0, k(x_p, x_q)) \]

- A function is said to be drawn from a Gaussian Process when any finite set of values follows a joint Gaussian distribution

- Kernel function: \( k(x_p, x_q) = \text{cov}(f(x_p), f(x_q)) \)
  - Governs nature (e.g., smoothness) of functions
  - Examples:
    \[ k(x_p, x_q) \propto \exp\left(-\frac{r^2}{2\ell^2}\right) \]
    \[ k(x_p, x_q) \propto \exp\left(-\frac{r}{\ell}\right) \]
    \[ r = \|x_p - x_q\|_2 \]
Gaussian Process Regression (GPR)

- Predictive distribution is also Gaussian

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>• No need to choose basis functions</td>
<td>• Computational expense due to matrix inversion</td>
</tr>
<tr>
<td>• Mathematically tractable</td>
<td></td>
</tr>
<tr>
<td>• Fast estimation of full predictive distribution</td>
<td></td>
</tr>
</tbody>
</table>
GPR models of takeoff weight

- Based on feature selection, we wish to determine TOW models of the form

\[ m = m(R, \rho, V_1, V_2) \]

where \( R, \rho, V_1, \) and \( V_2 \) are the observed inputs

- For the test dataset, given a set of input conditions, we would like to determine the predictive distribution of the corresponding output (i.e., the TOW)

- Flight Data Recorder (FDR) archives can be leveraged in model development, since they contain ground-truth data (the actual TOWs)
Identifying and validating takeoff weight models

- Divide data for each aircraft type into training (65% of flights), validation (15%), and test (20%) sets
- GPR models developed using training sets, and selected using validation sets
- Performance metrics presented for independent test sets
- Resulting TOW predictive distributions are also Gaussian
  - Used to calculate mean TOW and 95% prediction intervals
## Aircraft and engine types

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Engine type</th>
<th># flights</th>
<th>OEW (kg)</th>
<th>MTOW (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airbus A319-112</td>
<td>2 x CFMI CFM56-5B6/2, 2P</td>
<td>130</td>
<td>40,800</td>
<td>75,500</td>
</tr>
<tr>
<td>Airbus A320-214</td>
<td>2 x CFMI CFM56-5B4/2, P, 2P</td>
<td>169</td>
<td>42,600</td>
<td>77,000</td>
</tr>
<tr>
<td>Airbus A321-111</td>
<td>2 x CFMI CFM56-5B1/2, 2P</td>
<td>117</td>
<td>48,500</td>
<td>89,000</td>
</tr>
<tr>
<td>Airbus A330-202</td>
<td>2 x GE CF6-80E1A4</td>
<td>84</td>
<td>120,500</td>
<td>217,000</td>
</tr>
<tr>
<td>Airbus A330-243</td>
<td>2 x RR Trent 772B-60</td>
<td>100</td>
<td>120,500</td>
<td>233,000</td>
</tr>
<tr>
<td>Airbus A340-541</td>
<td>4 x RR Trent 553</td>
<td>52</td>
<td>170,400</td>
<td>365,000</td>
</tr>
<tr>
<td>Boeing B767-300</td>
<td>2 x GE CF6-80C2B7F</td>
<td>91</td>
<td>86,100</td>
<td>159,000</td>
</tr>
<tr>
<td>Boeing B777-300ER</td>
<td>2 x GE90-115B1</td>
<td>131</td>
<td>167,800</td>
<td>351,500</td>
</tr>
</tbody>
</table>
Predictive performance metrics

- Mean Absolute Relative Prediction Error or Mean Error (ME):

\[
ME = \frac{1}{n^*} \sum_{i=1}^{n^*} \left| \frac{m_i - \hat{m}_i}{m_i} \right|
\]

- Normalized Root Mean Squared Prediction Error (RMSE):

\[
RMSE = \sqrt{\frac{1}{n^*} \sum_{i=1}^{n^*} (m_i - \hat{m}_i)^2} \div \text{sd}(\hat{m})
\]

- Prediction Coverage (PC): Percentage of observations in test dataset for which 95% prediction intervals include the actual TOW values
Aircraft Noise and Performance (ANP) database

- Part of FAA’s Aviation Environmental Design Tool (AEDT)

- Takeoff mass modeled as a piecewise constant function of the flight stage/trip length

- Flight stage/trip length determined by the great circle distance between origin and destination airports

- Deterministic model (point estimate)
## Predictive performance on test data

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Gaussian Process Regression</th>
<th>ANP</th>
<th>Model Comparison: p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ME (%)</td>
<td>RMSE</td>
<td>PC (%)</td>
</tr>
<tr>
<td>A319-112</td>
<td>4.57</td>
<td>1.99</td>
<td>96</td>
</tr>
<tr>
<td>A320-214</td>
<td>3.58</td>
<td>1.39</td>
<td>100</td>
</tr>
<tr>
<td>A321-111</td>
<td>6.15</td>
<td>1.80</td>
<td>96</td>
</tr>
<tr>
<td>A330-202</td>
<td>2.22</td>
<td>0.43</td>
<td>100</td>
</tr>
<tr>
<td>A330-243</td>
<td>1.85</td>
<td>0.32</td>
<td>95</td>
</tr>
<tr>
<td>A340-541</td>
<td>1.67</td>
<td>0.31</td>
<td>100</td>
</tr>
<tr>
<td>B767-300</td>
<td>1.93</td>
<td>0.34</td>
<td>100</td>
</tr>
<tr>
<td>B777-300(ER)</td>
<td>1.99</td>
<td>0.39</td>
<td>96</td>
</tr>
</tbody>
</table>
Application to trajectory surveillance data

- GPR model trained using FDR data from A330-300s (60 flights), departing from six US airports.
- Model applied to takeoff roll trajectory variables from FDR and ASDE-X to estimate TOW for 37 flights (independent test set).
- TOW estimates compared to ground-truth values in FDR.

Prediction input source

<table>
<thead>
<tr>
<th></th>
<th>ASDE-X</th>
<th>FDR</th>
<th>ANP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Error (%)</td>
<td>2.85</td>
<td>3.07</td>
<td>3.08</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.01</td>
<td>1.0</td>
<td>1.45</td>
</tr>
<tr>
<td>PC (%)</td>
<td>100</td>
<td>100</td>
<td>--</td>
</tr>
</tbody>
</table>

No statistically significant difference in performance of models with respect to predicting mean error.
Application to fuel flow rate modeling

- GPR can also be used to model the fuel flow rate in ascent.
- Predictor variables:
  - Dynamic pressure multiplied by wing area
  - TOW
  - Ratio of vertical speed to ground speed
  - Ground speed
  - Rate of change of ground speed

- More accurate estimates of TOW can improve estimates of fuel flow rate.
Fuel flow rate sensitivity to TOW

- 2% deviation in estimated TOW can result in an 8% increase in the mean error in estimated fuel flow rate (results based on A321-111 in ascent phase)
Using TOW model for fuel flow rate estimation

- **Model 1:** TOW estimated using GPR model
  - \( p(\dot{m}_f|\mathbf{x}_\mathbf{-m}, \phi, \mathcal{D}_1, \mathcal{D}_2) \) is a Gaussian Mixture PDF
    
    \[
    m \sim p(m|\phi, \mathcal{D}_2) \quad \text{(Gaussian under GPR)}
    \]
    
    \[
    p(\dot{m}_f|\mathbf{x}_\mathbf{-m}, \phi, \mathcal{D}_1, \mathcal{D}_2) = \int_m p(\dot{m}_f|\mathbf{x}_\mathbf{-m}, m, \mathcal{D}_1)p(m|\phi, \mathcal{D}_2)dm
    \]
    
    \[
    \approx \frac{1}{n_s} \sum_{i=1}^{n_s} p(\dot{m}_f|\mathbf{x}_\mathbf{-m}, m_i, \mathcal{D}_1) \quad \text{(Gaussian Mixture)}
    \]

- **Model 2:** TOW estimated using ANP
  - \( p(\dot{m}_f|\mathbf{x}_\mathbf{-m}, \phi, \mathcal{D}_1, \mathcal{D}_2) \) is a Gaussian PDF
Evaluation of a fuel flow rate model

- Using test data for the A321-111 in ascent

<table>
<thead>
<tr>
<th></th>
<th>Individual Model Performance</th>
<th>Model Comparison: p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>ME (%)</td>
<td>4.40</td>
<td>5.16</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td>PC (%)</td>
<td>93.78</td>
<td>80.15</td>
</tr>
</tbody>
</table>

- GPR-based TOW model [Model 1] yields an improved fuel flow rate model over the (deterministic) ANP TOW model [Model 2]
- Also consider GPR models where the TOW is not explicitly included (details in paper)
Summary

- Developed models that infer TOW from takeoff roll trajectory data
- Used Gaussian Process Regression methodology to build statistical models of TOW which also quantify uncertainty
  - Mean error found to be approximately 3% (averaged across eight aircraft/engine types)
  - Nearly 50% reduction in mean error, relative to ANP
- Demonstrated how proposed models can be used with the availability of other surveillance sources (e.g., ASDE-X)
- Demonstrated the use of proposed statistical TOW models for fuel flow rate estimation