An optimization model for assigning 4D-trajectories to flights under the TBO concept

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Presentation Content

1. OptiFrame Project background
2. Mathematical model
3. Solution approaches
4. Computational Experience
5. Concluding Remarks
OptiFrame Project Objectives (1/2)

- SESAR ER project

- Application of principles of mathematical modelling and optimization to configure and assess the performance of the Trajectory Based Operations (TBO) concept.
  - Viability of the concept
  - Major issues (e.g., barriers, constraints, stakeholders’ expectations, etc.)
  - Whether and to what extent the objectives of flexibility of airspace users and predictability of the ATM system can be achieved
OptiFrame Project Objectives (2/2)

Research questions

• What are the barriers, enablers and potential benefits from the TBO concept implementation?
• How stakeholders priorities and preferences can be incorporated into mathematical models?
• What are the scalability issues associated with the development and implementation of these models and algorithms?
Expected Project Outcome

Development of a framework to address the identified objectives and provide guidance to decision makers on the potential and conditions of deployment of the TBO concept.

This framework, consisting of mathematical models, solution algorithms and conditions of applicability, can be used:

1. as a “simulator” to fully understand the benefits and limitations of the TBO approach;
2. to investigate trade-offs between different competing objectives relevant to the TBO concept;
3. It explicitly takes into account the route preferences and flight criteria and priorities of the airspace users.
Trajectory Based Operations

Air traffic management (ATM) considers the trajectory of a manned or unmanned vehicle during all phases of flight and manages the interaction of that trajectory with other trajectories or hazards to achieve the optimum system outcome, with minimal deviation from the user-requested flight trajectory, whenever possible. (ICAO Doc. 9854, §1.9.2 )
Preferences

• OptiFrame Stakeholders’ Workshop (Brussels October 5th, 2016)

• Preferences

• Priorities

• **Major Performance Determinants:**
  • Delays
  • 2D routes (reroute)
  • Flight levels
Airspace users’ priorities

SESAR

UDPP

UDPP Step 1

Slot swapping

UDPP Step 2

Fleet Delay Apportionment (FDA)

Selective Flight Protection (SFP)
Multi-objective approach

OptiFrame modelling approach

Decision-making context

- Single decision-maker or decision group
- Decision-making context
  - Bottom-up information flow
    - Model that generates non-dominated solutions
  - Top-down information flow
    - Model that incorporates preferences

Conflict resolution

Models for multiple-decision-makers

Overview of ATFM models:

Trajectory definition model:

☉ “Considers” the preferred 4D-trajectory of all the flights

☉ Output: 4D-trajectory for each flight.

☉ The output trajectories are obtained by minimising the deviation from the original preferred trajectories

☉ Incorporate stakeholders’ preferences and priorities

Trajectory selection model:

☉ Considers that each flight is submitted to the system with a set of alternative trajectories.

☉ The model aims at selecting the flight trajectories that optimise the efficiency of the ATM system.

☉ Incorporate stakeholders’ preferences and priorities
Literature review

For the first type of models

Helme (1992)
Lindsay et al. (1993)
Bertsimas & Stock (1998)
Bertsimas & Stock Patterson (2000)
Bertsimas, Lulli Odoni (2011)
H. Balakrishnan and B. Chandran (2014)

For the second type of models

Sherali, Smith, Trani (2006)
Trajectory model

We represent the airspace as a directed graph \( G = (\mathcal{N}; \mathcal{E}) \) in the 2D-space.
We represent the airspace as a directed graph $G = (\mathcal{N}; \mathcal{E})$ in the 2D-space.
Decision variables:

\[ x_{e,l}^f(t) = \begin{cases} 
1 & \text{if flight } f \text{ is planned to travel on arc } e \in \mathcal{E}_f \\
0 & \text{at flight level } l \in \mathcal{L}_f \text{ by time } t, \\
\end{cases} \]

otherwise.
Objectives

Obj 1. Minimize departure delay

\[
\sum_{f \in F} \sum_{e \in \Delta_f^+} \sum_{t \in T_f} (t - t_f) \left( x^f_e(t) - x^f_e(t - 1) \right)
\]

Obj 2. Minimize deviation from the preferred 3D routes

\[
\sum_{f \in F} \left( \sum_{e \in \mathcal{E}_f, l \in \mathcal{L}_f} \bar{C}^f_{e,l} \cdot x^f_{e,l} \left( \overline{T^f_e} \right) - C^*_f \right)
\]

Obj 3. Minimize the ANS route charges

\[
\sum_{f \in F, s \in S, e \in \mathcal{E}_f, l \in \mathcal{L}_f} R^f_s \cdot x^f_{e,l} \left( \overline{T^f_e} \right)
\]
Constraints

1. Each aircraft flies one 4D-trajectory
   • Continuity (time, arcs used)
   • Physical feasibility (Flying levels)

2. Airports and en-route sectors are capacitated

3. Flights’ prioritisation
Continuity

A flight must arrive at one of the subsequent arcs by at most $\alpha_{f,e}^+$ time units (the maximum possible) after traveling through the preceding arc:

$$\sum_{e \in \Delta_{f,n}^-} x_{e,l}^f (t - \alpha_{f,e}^+) \leq \sum_{e \in \Delta_{f,n}^+} x_{e,l'}^f (t)$$

A flight cannot enter the next arc (sector) on its path until it has spent at least $\alpha_{f,e}^-$ time units (the minimum possible) traveling through one of the preceding arcs (sectors) on its current path:

$$\sum_{e \in \Delta_{f,n}^+} x_{e,l}^f (t - \alpha_{f,e}^-) \geq \sum_{l \in L_f, e \in \Delta_{f,n}^+} x_{e,l}^f (t)$$
Capacity constraints

- The number of flight **departures** from airport $k$ at time period $t$ does not exceed its **departure capacity** during that time period:

\[
\sum_{f \in F : k \equiv d, e \in \Delta^+_f,k} \left( x_{e,t}^f(t) - x_{e,t}^f(t-1) \right) \leq D^t_k
\]

- The number of flight **arrivals** at airport $k$ at time period $t$ does not exceed its **arrival capacity** during that time period:

\[
\sum_{f \in F : k \equiv a, e \in \Delta^-_f,k} \left( x_{e,t}^f(t) - x_{e,t}^f(t-1) \right) \leq A^t_k
\]

- The number of flights **entering the en-route sector** $s$ at time period $t$ does not exceed its **capacity of the sector** during that time period:

\[
\sum_{f \in F, l \in L_f, e \in I_s} \left( x_{e,t}^f(t) - x_{e,t}^f(t-1) \right) \leq E^t_s
\]
Fleet Delay Apportionment (FDA) mechanism

1. AU assigns priority value $\tau_f$ to flight $f$ (between 1-9)

2. The system calculates the max delay of flight $f$:

$$\gamma_f = \left( \frac{\tau_f \cdot \sigma_f}{\sum_{f \in \Psi} \tau_f \cdot \sigma_f} \right) \cdot \zeta_{\varphi}$$

3. $\gamma_f$ is input to our model to define a delay (prioritisation constraint)

- $\sigma_f$ is the baseline delay of flight $f$
- $\Psi$ is the set of all the airspace users, with an AU being $\varphi \in \Psi$,
- $\zeta_{\varphi} = \sum_{f \in \Psi} \sigma_f$, $\varphi \in \Psi$ is the total amount of delay to be absorbed by AU $\varphi$ in the absence of prioritization, with $\Psi_{\varphi}$ being...
Flight prioritization (FDA)

These constraints ensure that the total delay assigned to each flight does not exceed the maximum amount of delay $\gamma_f$ to be absorbed by that flight according to the AU’s priority point.

\[ x_{e',l_0}^f(t) - x_{e,l_0}^f(t + \bar{t}_f - t_f + \gamma_f) \leq 0 \]

\[ \gamma_f = \left( \frac{\tau_f \cdot \sigma_f}{\sum_{f \in \mathcal{F}_\varphi} \tau_f \cdot \sigma_f} \right) \cdot \zeta_\varphi \]
Summary

\[ \min \begin{cases} 
1) & \text{delay} \\
2) & \text{deviation from user preferred route} \\
3) & \text{ANS route charge (AUs concern)} 
\end{cases} \]

1. The single trajectory constraints

2. The airports and en-route sectors capacity constraints

3. Delay constraints (prioritisation)
Exact approach

- Each MIP is solved using the IBM Cplex MIP solver

- First we solve the single objective to see if there really is a trade-off between the three objectives

- Secondly, we solve the multi-objective optimisation to compute the non-dominated solutions

- The non-dominated solutions are computed using the Quadrant Shrinking Method, Boland et al. (2016).
### Single objectives (1/2)

<table>
<thead>
<tr>
<th>Objective</th>
<th>$\hat{x}_{\text{delay}}$</th>
<th>$\hat{x}_{\text{deviation}}$</th>
<th>$\hat{x}_{\text{RoadCharge}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min delay (%)</td>
<td>0.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Min deviation (%)</td>
<td>100.00</td>
<td>0.00</td>
<td>19.00</td>
</tr>
<tr>
<td>Min road charges (%)</td>
<td>100.00</td>
<td>77.96</td>
<td>0.00</td>
</tr>
<tr>
<td>Time (s)</td>
<td>600.72</td>
<td>364.47</td>
<td>288.39</td>
</tr>
</tbody>
</table>

- 2000 flights
- 50 airports
- 150 sectors
- 96 time periods
### Single objectives (2/2)

<table>
<thead>
<tr>
<th></th>
<th>( \hat{x}_{\text{delay}} )</th>
<th>( \hat{x}_{\text{deviation}} )</th>
<th>( \hat{x}_{\text{RoadCharges}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min delay (%)</td>
<td>0.00</td>
<td>99.97</td>
<td>100.00</td>
</tr>
<tr>
<td>Min deviation (%)</td>
<td>100.00</td>
<td>0.00</td>
<td>18.91</td>
</tr>
<tr>
<td>Min road charges (%)</td>
<td>100.00</td>
<td>74.22</td>
<td>0.00</td>
</tr>
<tr>
<td>Time (s)</td>
<td>1195.29</td>
<td>600.03</td>
<td>417.92</td>
</tr>
</tbody>
</table>

- 2000 flights
- 50 airports
- 150 sectors
- 96 time periods
Tri-objective (2/3)

- 100 flights
- 5 airports
- 20 sectors
- 25 time periods
Tri-objective (3/3)

- 2000 flights
- 50 airports
- 150 sectors
- 96 time periods
Heuristic algorithm: “Local Search” (1/3)

- Preferred trajectories
- Find a feasible time schedule solving an Integer program

3 possible neighbours:
1. Modify the flight levels
2. 1-waypoint reroute
3. 2-waypoints reroute
Heuristic algorithm: “Local Search” (2/3)

Preliminary results on randomly generated instances:
Instance A: 50 sectors, 13 airports, 117 flights
Computational time: 7200 seconds (time limit reached)

- Flight to be rerouted is randomly chosen
- Red: non-dominated solutions
- Flight to be rerouted is the most delayed
- Orange: non-dominated solutions
- Jointly non-dominated solutions are circled in blue
Heuristic algorithm: “Local Search” (3/3)

Preliminary results on randomly generated instances:
Instance B: 100 sectors, 28 airports, 688 flights
Computational time: 7200 seconds (time limit reached)

- Flight to be rerouted is randomly chosen.
- Red: non-dominated solutions
- Flight to be rerouted is the most delayed.
- Orange: non-dominated solutions
- Jointly non-dominated solutions are circled in blue
Concluding Remarks

• The proposed framework can be used:

  1. to investigate trade-offs between different competing objectives relevant to the TBO concept;
  2. as a “simulator” to understand the benefits and limitations of the TBO approach.

• Ongoing work: further experimentation and improvement of the proposed algorithms.
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OptiFrame Project

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