Modeling Airspace Stability and Capacity for Decentralized Separation

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Outline

Introduction

Analytical Stability-Capacity Model

Fast-Time Simulation Experiments

Results

Conclusions
1. Introduction
Demand vs. Capacity

+2.4% Traffic Demand
+20.9% En Route Delays
+25.3% Serious Separation Minima Infringements

ATC Capacity (ERT)
ATC Staffing (ERT)
Weather (ERT)
ATC Disruptions (ERT)
Events (ERT)
Reroutings (ERT)
Disruptions (ERT)

En-route ATFM delays (million minutes)

2015 result

55.3% € 479 million

#Eurocontrol PRR 2016
Centralized vs. Decentralized Airspace

Decentralized ATC is expected to **increase capacity**

[Adapted from J. M. Hoekstra 2001]
What is Capacity Decentralized ATC?

- Air traffic controller *workload not relevant* for decentralization
- Capacity is density at which airspace becomes *saturated*
- But when is the airspace saturated?

Airspace Stability
Intrusions vs. Conflicts

- Intrusions/loss-of-separations occur when the minimum separation requirements are violated.
- Conflicts are predicted intrusions within the look-ahead time.
Airspace Stability

<table>
<thead>
<tr>
<th>#</th>
<th>Conflict</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>A-B</td>
</tr>
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</table>
Airspace Stability

<table>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>A-B</td>
<td>Primary</td>
</tr>
<tr>
<td>2</td>
<td>A-C</td>
<td>Secondary</td>
</tr>
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<td>2</td>
<td>A-C</td>
<td>Secondary</td>
</tr>
<tr>
<td>3</td>
<td>C-B</td>
<td>Secondary</td>
</tr>
</tbody>
</table>

Conflict Chain Reactions

Domino Effect Parameter

#K. Bilimoria et al 2000
[J. Krozel et al 2000]
Domino Effect Parameter (DEP) #

\[ DEP = \frac{R_3 - R_1}{R_1 + R_2} = \frac{C_{total_{wr}}}{C_{total_{nr}}} - 1 \]

Where:
\[ C_{total_{nr}} = \text{Total conflicts no resolution} \]
\[ C_{total_{wr}} = \text{Total conflicts with resolution} \]

DEP is the number of secondary conflicts per primary conflict

# [K. Bilimoria et al 2000]
# [J. Krozel et al 2000]
DEP Example

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<td>Secondary</td>
</tr>
<tr>
<td>3</td>
<td>C-B</td>
<td>Secondary</td>
</tr>
</tbody>
</table>
**DEP Example**

\[ DEP = \frac{3}{1} - 1 = 2 \]

Higher DEP → Lower Stability → Lower Capacity
Stability and Capacity: Previous Research

- $DEP = \frac{c_{totalWR}}{c_{totalnr}} - 1 \approx \frac{\rho}{\rho_{max} - \rho}$

- $\rho_{max} = \frac{1}{D_{sep}k_{cdr}p_s}$

- Capacity definition: If $\rho \equiv \rho_{max}$ then $DEP \to \infty$

- Limitations:
  - Semi-empirical method
  - Requires time-consuming simulation experiments

Where:

- $\rho$ = Traffic Density
- $\rho_{max}$ = Max Density / Capacity
- $D_{sep}$ = Separation Requirement
- $k_{cdr}$ = Effect of CD&R on stability
- $p_s$ = Effect of route structure on instantaneous conflict probability

# [M. Jardin 2004, 2005]
Goal of this Research

• Extend the previous semi-empirical method to an **analytical capacity model**

• Understand the effect of **physical airspace parameters** on capacity
  – Separation minima, look-ahead time, area …

• **Scope:**
  – For **2D direct-routing airspace** (only cruising aircraft)
  – **Ideal conditions**

• **Application:**
  – **First order estimate** of capacity
  – Determine which cases to further explore using simulations
2. Analytical Stability-Capacity Model
Modeling Approach

\[ DEP = \frac{C_{total_{wr}}}{C_{total_{nr}}} - 1 \]

Conflicts without CR → Conflicts with CR → Capacity
Modeling Approach

\[ DEP = \frac{C_{total_{wr}}}{C_{total_{nr}}} - 1 \]
Modeling Conflicts Without CR

Global Instantaneous Conflict Rate

\[
C_{ssnr} = \frac{N(N-1)}{2} p_2
\]

\[
p_2 = \frac{A_c}{A} = \frac{2 D_{sep} v_{rel} t_l}{A}
\]

\[
v_{rel} = \frac{4 v}{\pi}
\]

Where:

- \( C_{ssnr} \) = Global conflict rate without CR
- \( N \) = Number of instantaneous aircraft
- \( p_2 \) = Conflict probability between any 2 aircraft without CR
- \( A_c \) = Area searched for conflicts
- \( A \) = Total airspace area
- \( D_{sep} \) = Separation minima
- \( v_{rel} \) = Relative Velocity
- \( v \) = Ground speed of aircraft
- \( t_l \) = Conflict detection look-ahead time

#[J. M. Hoekstra et al 2016]
Modeling Conflicts Without CR

Total Number of Conflicts Without CR

- \( C_{total_{nr}} = \frac{1}{t_{cnr}} \int_0^T C_{ss_{nr}} \, dt \)
- \( C_{total_{nr}} = \frac{C_{ss_{nr}} T}{t_{cnr}} \)
- \( t_{cnr} = \frac{T \, t_l - \frac{1}{2} t_l^2}{T} \)
- \( N = \rho \, A \)
- \( C_{total_{nr}} = \frac{2 \, D_{sep} \, \nu_{rel} \, T^2 \, \rho \, A \left( \rho - \frac{1}{A} \right)}{2 \, T - t_l} \)

Finite Time Measurements

Where:
- \( C_{total_{nr}} = \) Total conflicts without CR
- \( t_{cnr} = \) Average conflict duration
- \( T = \) Analysis time
- \( \rho = \) Traffic density
Modeling Conflicts Without CR

Local Conflict Rate Without CR

- \( r_{cnr} = \frac{C_{1nr}}{L_{nr}} \)
- \( C_{1nr} = \frac{C_{totalnr}}{N_{totalnr}} \)
- \( N_{totalnr} = N + \frac{N v}{L_{nr}} T \)
- \( r_{cnr} = \frac{2 D_{sep} v_{rel} T^2 (\rho -1/A)}{(2 T - t_i) (v T + L_{nr})} \)

Where:

- \( r_{cnr} \) = Local conflict rate per unit distance without CR
- \( C_{1nr} \) = Total conflicts for 1 aircraft without CR
- \( L_{nr} \) = Average total distance without CR
- \( N_{totalnr} \) = Total aircraft number without CR
Modeling Approach

\[ DEP = \frac{C_{total\,wr}}{C_{total\,nr}} - 1 \]

- Conflicts without CR
- Effect of Direct Routing & Finite Time Measurements
- Conflicts with CR
- Capacity
- Effect of CD&R on Stability
Modeling Conflicts With CR

- \( C_{1wr} = r_{cwr} \left( L_{nr} + k_{cdr} \ C_{1wr} \right) \)

- \( C_{1wr} = \frac{L_{nr} \ r_{cwr}}{1 - k_{cdr} \ r_{cwr}} \)

- \( C_{total wr} = C_{1wr} \ N_{total wr} \)

Where:
- \( r_{cwr} \) = Local conflict rate per unit distance with CR
- \( C_{1wr} \) = Total conflicts for 1 aircraft with CR
- \( k_{cdr} \) = Extra distance searched due to CD&R
- \( N_{total wr} \) = Total aircraft number with CR
- \( C_{total wr} \) = Total conflicts with CR
Modeling Conflicts With CR

Assumptions to relate conflicts without and with CR

1. $N_{\text{total wr}} \approx N_{\text{total nr}}$
   - $N_{\text{total nr}} = N + \frac{N v}{L_{\text{nr}}} T$
   - $N$ and $L_{\text{nr}}$ are both expected to increase by a proportional amount with CR

2. $r_{\text{c wr}} \approx r_{\text{c nr}}$
   - No preferred directions with or without CR for direct routing
   - Distribution of conflict angles not expected to change significantly with and without CR
Modeling Conflicts With CR

\[ C_{total_{wr}} = \frac{N_{total_{wr}} L_{nr} r_{cwr}}{1 - k_{cdr} r_{cwr}} \]

• Assume:
  
  – \( N_{total_{wr}} \approx N_{total_{nr}} \)
  – \( r_{cwr} \approx r_{c_{nr}} \)

\[ C_{total_{wr}} = \frac{2 \rho A D_{sep} v_{rel} T^2 (\rho - 1/A) (v T + L_{nr})}{(2T - t_l) (v T + L_{nr}) - 2 k_{cdr} D_{sep} v_{rel} T^2 (\rho - 1/A)} \]
Modeling Approach

\[ \text{DEP} = \frac{C_{\text{total wr}}}{C_{\text{total nr}}} - 1 \]

- Conflicts without CR
- Effect of Direct Routing & Finite Time Measurements
- Conflicts with CR
- +
- Capacity
- Effect of CD&R on Stability
Effect of CD&R on Stability

\[ k_{cadr} = k_{cd} + k_{cr} \]
Effect of CD&R on Stability

Extra Distance Searched Due to CD (State-Based)

\[ k_{cd} = v_{rel} t_l \]
Effect of CD&R on Stability

Extra Distance Searched Due to CR (MVP)

\[ \vec{dV}_{mvp}(\theta, D_{cpa}) = \frac{D_i}{t_{cpa}} \frac{\vec{D}_{cpa}}{D_{cpa}} \]

Where:
- \( \vec{dV}_{mvp} \) = MVP commanded CR vector
- \( D_i \) = Intrusion distance (scalar)
- \( t_{cpa} \) = Time to Closeset Point of Approach (CPA)
- \( \vec{D}_{cpa}/D_{cpa} \) = Unit vector from intruder to CPA

# [Adapted from J. Ellerbroek 2013]
Effect of CD&R on Stability
Extra Distance Searched Due to CR (MVP)

\[ k_{cr} (\theta, D_{cpa}) = | \overrightarrow{dv_{mvp}} (\theta, D_{cpa}) | t_{cpa} \]

Baseline Experiment
\[ D_{sep} = 2.5 \text{ NM} \]
\[ t_I = 5 \text{ mins} \]
\[ v = 550 \text{ kts} \]
Effect of CD&R on Stability

Extra Distance Searched Due to CR (MVP)

\[
\bar{k}_{cr} = \frac{\int_0^{D_{sep}} \int_0^\pi k_{cr} W_\theta W_{D_{cpa}} \, d\theta \, dD_{cpa}}{\int_0^{D_{sep}} \int_0^\pi W_\theta W_{D_{cpa}} \, d\theta \, dD_{cpa}}
\]

\[W_\theta = \text{P.D.F. of conflict angles} \rightarrow \text{Triangular for direct routing} \]
\[W_{D_{cpa}} = \text{P.D.F. of distance to CPA} \rightarrow \text{Uniform for direct routing}\]

Baseline Experiment

\[D_{sep} = 2.5 \text{ NM} \]
\[t_l = 5 \text{ mins} \]
\[v = 550 \text{ kts} \]
\[\bar{k}_{cr} = 0.84 \text{ NM} \]
\[k_{cd} = 58.4 \text{ NM} \]

# [J. M. Hoekstra et al 2016]
Modeling Approach

\[ DEP = \frac{C_{total\ wr}}{C_{total\ nr}} - 1 \]

- Conflicts without CR
- Effect of Direct Routing & Finite Time Measurements
- Conflicts with CR
- Capacity
- Effect of CD&R on Stability
Modeling Capacity

- \( C_{total_{nr}} = \frac{2 D_{sep} v_{rel} T^2 \rho A (\rho - 1/A)}{2 T - t_l} \)

- \( C_{total_{wr}} = \frac{2 \rho A D_{sep} v_{rel} T^2 (\rho - 1/A)(v T+L_{nr})}{(2T - t_l)(v T+L_{nr})} - 2 k_{cdr} D_{sep} v_{rel} T^2 (\rho - 1/A) \)

- \( v_{rel} = \frac{4 v}{\pi} \)

- \( k_{cdr} = k_{cd} + \bar{k}_{cr} = v_{rel} t_l + \frac{\int_{0}^{D_{sep}} \int_{0}^{\pi} k_{cr} W_\theta W_{Dcpa} d\theta dD_{cpa}}{\int_{0}^{D_{sep}} \int_{0}^{\pi} W_\theta W_{Dcpa} d\theta dD_{cpa}} \)

\[ DEP = \frac{C_{total_{wr}}}{C_{total_{nr}}} - 1 \]
Modeling Capacity

• \(DEP = \frac{C_{total\text{wr}}}{C_{total\text{nr}}} - 1 \approx \frac{\rho}{\rho_{max} - \rho}\)

• Capacity definition: If \(\rho \equiv \rho_{max}\) then \(DEP \to \infty\)

\[
\rho_{max} = \frac{(v T + L_{nr})(2T - t_l)}{2 k_{cdr} D_{sep} v_{rel} T^2}
\]

Where:

- \(DEP\) = Domino Effect Parameter
- \(C_{total\text{wr}}\) = Total conflicts with CR
- \(C_{total\text{nr}}\) = Total conflicts without CR
- \(\rho\) = Traffic Density
- \(v\) = Ground speed (equal)
- \(v_{rel}\) = Relative velocity
- \(T\) = Total analysis time
- \(L_{nr}\) = Average flight distance without CR
- \(t_l\) = CD look-ahead time
- \(D_{sep}\) = Separation minima
- \(k_{cdr}\) = Effect of CD&R on stability
3. Fast-Time Simulation Experiments
BlueSky Open ATM Simulator

https://github.com/ProfHoekstra/bluesky
Experiment Physical Area

Scenario Properties

- Aircraft required to cross sector
- Equal Airspeed
- Constant density
- Uniform heading distribution
- Aircraft fly parallel trajectories after CR
- Aircraft deleted when leaving sector
Independent Variables

1. Traffic Demand

<table>
<thead>
<tr>
<th>#</th>
<th>Density [ac/10,000 NM²]</th>
<th>Number of Instantaneous AC [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>1.51</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>2.27</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>3.42</td>
<td>86</td>
</tr>
<tr>
<td>5</td>
<td>5.15</td>
<td>129</td>
</tr>
<tr>
<td>6</td>
<td>7.76</td>
<td>194</td>
</tr>
<tr>
<td>7</td>
<td>11.70</td>
<td>292</td>
</tr>
<tr>
<td>8</td>
<td>17.62</td>
<td>440</td>
</tr>
<tr>
<td>9</td>
<td>26.55</td>
<td>664</td>
</tr>
<tr>
<td>10</td>
<td>40.00</td>
<td>1000</td>
</tr>
</tbody>
</table>

- Europe: ≈ 7.5 per 10,000 NM² in Class A (FR24 14:00 6/14/2017)

- 10 densities x 3 CD&R settings x 10 repetitions x 2 CR (OFF/ON) = 600 Runs

2. CD&R Settings

<table>
<thead>
<tr>
<th>Condition Name</th>
<th>Separation Minimum [NM]</th>
<th>Look-Ahead [mins]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>2.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Half Look-Ahead</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Double Separation</td>
<td>5.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

>550,000 flights simulated
Dependent Variables

**Least-square fit** of the simulation data to:

1. Global conflict rate *without* CR
2. Total number of conflicts *without* CR
3. Total number of aircraft *with* and *without* CR
4. Local conflict rate per unit distance *with* and *without* CR
5. Capacity (DEP model)

\[
\text{Accuracy} = 100\% - \frac{|\varepsilon_{\text{model}} - \varepsilon_{\text{LQ fit}}|}{\varepsilon_{\text{LQ fit}}}\%
\]
4.

Results
Global Conflict Rate Without CR

\[ C_{ssnr} = \frac{N(N - 1)}{2} p_2 \]

\[ p_2 = \frac{2 D_{sep} v_{rel} t_I}{A} \]

**Baseline**

\( p_2 \) Accuracy = 90.59%

**Half Look-Ahead**

\( p_2 \) Accuracy = 93.51%

**Double Separation**

\( p_2 \) Accuracy = 90.33%
Total Conflict Number Without CR

Baseline

\[ \epsilon_{model} \text{ Accuracy} = 87.81\% \]

\[ C_{total_{nr}} = \frac{2 D_{sep} v_{rel} T^2 \rho A (\rho - 1/A)}{2T - t_l} \epsilon_{fit} \]

Half Look-Ahead

\[ \epsilon_{model} \text{ Accuracy} = 90.16\% \]

Double Separation

\[ \epsilon_{model} \text{ Accuracy} = 87.95\% \]
**Total Aircraft Count With and Without CR**

\[ N_{total_{nr}} = \left( N + \frac{N \nu}{L_{nr}} \right) \epsilon_{fit} \]

**Assumption:**

\[ N_{total_{wr}} \approx N_{total_{nr}} \]

---

**Baseline**

- \( \epsilon_{model} \text{ Accuracy} = 93.45\% \)
- \( \epsilon_{model} \text{ Accuracy} = 93.38\% \)
- \( \epsilon_{model} \text{ Accuracy} = 93.38\% \)
- \( \epsilon_{model} \text{ Accuracy} = 93.46\% \)

**Half Look-Ahead**

- \( \epsilon_{model} \text{ Accuracy} = 93.46\% \)

**Double Separation**

- \( \epsilon_{model} \text{ Accuracy} = 93.58\% \)
- \( \epsilon_{model} \text{ Accuracy} = 93.38\% \)
Local Conflict Rate With and Without CR

Baseline

\[ r_{cnr} = \frac{2 D_{sep} v_{rel} T^2 (\rho - 1/A)}{(2 T - t_l) (v T + L_{nr})} \epsilon_{fit} \]

Assumption:
\[ r_{cw} \approx r_{cnr} \]

Half Look-Ahead

\[ \epsilon_{model} \text{ Accuracy} = 76.23\% \]
\[ \epsilon_{model} \text{ Accuracy} = 94.1\% \]

Double Separation

\[ \epsilon_{model} \text{ Accuracy} = 86.4\% \]
\[ \epsilon_{model} \text{ Accuracy} = 96.5\% \]

\[ \epsilon_{model} \text{ Accuracy} = 70.47\% \]
\[ \epsilon_{model} \text{ Accuracy} = 94.05\% \]
Capacity (DEP Model)

\[
DEP \approx \frac{\rho}{\rho_{\text{max}} - \rho}
\]

\[
\rho_{\text{max}} = \frac{(vT + L_{\text{nrr}})(2T - t_l)}{2\ k_{\text{dcr}}\ D_{\text{sep}}\ v_{\text{rel}}\ T^2}
\]

Half Look-Ahead

- Model: 211.38
- Fit: 271.93
- Accuracy: 78.08%

Double Separation

- Model: 51.72
- Fit: 123.41
- Accuracy: 41.91%

Baseline

- Model: 105.38
- Fit: 154.11
- Accuracy: 68.38%
Analysis of Double Separation Experiment

Density Contours

CR OFF  CR ON

Baseline

Half

Look-Ahead

Double Separation

CR ON reduced density for Double Separation
Analysis of Double Separation Experiment

Distance to Destination (CR ON)

- CR ON ‘bounced’ aircraft out of the simulation, reducing flight distance
- Unanticipated reduction of flight distance \((L)\) led to an unanticipated increase of local conflict rate with CR ON → effect most pronounced for Double Separation
- Therefore simulation design partially responsible for lower than expected accuracy of CR ON models (including capacity)

Assumption:

\[
r_{cwr} \approx r_{cnr}
\]

\[
r_{cwr} \approx \frac{2 D_{sep} v_{rel} T^2 (\rho - 1/A)}{(2 T - t_l) (v T + L)}
\]
Conclusions
Conclusions

• The **theoretical** capacity limit of decentralized airspace can be defined as the density at which the Domino Effect Parameter, a measure of airspace stability, approaches infinity.

• An **analytical model** for capacity using this definition was derived as a function of airspace and CD&R parameters.

• Accuracy of the **CR OFF models was high**.

• Accuracy of the CR ON models was lower than expected due to the ‘**bouncing-out’ effect**
  – Hypothesis: Improve aircraft deletion criterion of simulation.

• Accuracy of model sufficient to gain a **first order estimate** of the maximum theoretical capacity.
  – Reasonable accuracy for conditions where ‘bouncing-out’ was small.
Future Work

\[ v_{rel} = \frac{4v}{\pi} \]

Conflict Rate in 3D + Airspace Structure

12:00 Safety and Resilience Track
Thankyou For Your Attention!

[e.sunil@tudelft.nl]
[https://www.researchgate.net/profile/Emmanuel_Sunil]
6.

Backup Slides
Theoretical vs. Practical Capacity

• In practice capacity is affected:
  – Weather
  – Airline economics
  – Mix of aircraft types
  – Hardware failure rates …

• Public will not accept asymptotic behavior of safety as a capacity limit

• Practical capacity hard to quantify
  – Workload varies from ATCo to ATCo

• But, the theoretical capacity definition is a useful and unbiased benchmark:
  – Which parameters affect capacity the most?
  – Which conditions should be analyzed using simulations?
Distance to Destination (CR ON)

Baseline

Half Look-Ahead

Double Separation
Effect of CD&R on Stability

Extra Distance Searched Due to CR (MVP)

- $\vec{v}_o = [v, 0]^T$
- $\vec{v}_i = [v \cos(-\theta), v \sin(-\theta)]^T$
- $\vec{v}_{rel} = \vec{v}_o - \vec{v}_i$
- $D_{rel} = t_l |\vec{v}_{rel}| + \sqrt{D_{sep}^2 - D_{cpa}^2}$
- $\vec{D}_{rel} = \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|} D_{rel}$
- $\vec{D} = \begin{bmatrix} D_{rel} & D_{cpa} \\ -D_{cpa} & D_{rel} \end{bmatrix} \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}$
- $\vec{D}_{cpa} = \vec{D}_{rel} - \vec{D}$
- $t_{cpa} = \frac{D_{rel}}{|\vec{v}_{rel}|}$
- $D_i = D_{sep} - D_{cpa}$

Main Variables:
- Conflict Angle, $\theta$
- Distance to intruder at CPA, $D_{cpa}$