European airspace

30,000 daily flights

Close to capacity limit

Managed at the country level

Traffic likely to double by 2020

⇒ Big challenge
Single European Sky

• Aiming at unifying the European Network

• Common rules for the European Air Traffic Management

• Meet future capacity and safety needs
Research program

- Single European Sky ATM Research Programme (SESAR)
- Launched in 2007
- Jointly managed by the European Commission and EUROCONTROL (European Organisation for the Safety of Air Navigation)
- Fundamental, applied and industrial research
- Target number: reduce by 10% the environmental impact and flight time
Sectorization

Grouped to be managed by Air Traffic Control
Maximum number of flights for each controller
Prediction of the number of flights is very important (see [Lulli and Odoni, 2007])
Air Traffic Control

Data available: expected number of flights

Actions taken based on experience of the ATC

- Demand (Traffic) (Occupancy counts)
- Capacity Acceptable # of A/C
- Regulations STAMs
- Sectorization
- Action on Demand
- Actions on Capacity
- Cost of delays measures staffing controller workload
Goal

Increase the average occupancy

Knowledge of the full distribution would allow to increase the average occupancy or to reduce the sector congestion.
COPTRA
Combining probable trajectories

COPTRA aims to propose an efficient method to build probabilistic traffic forecasts on the basis of flight trajectory predictions.

Define uncertainty in trajectories → Build Probabilistic Traffic Prediction → Apply to ATM tools → Improve Trajectory Prediction to Improve Traffic Performance

Partners: EUROCONTROL, CRIDA, BR&TE, ITU and UCL
Outlook

• Quantification of sector congestion

• Criticality measures on flights

• Reduce network congestion
Steps to evaluate overload

Data: stochastic trajectories

Probability to be in a sector in a given time

Distribution of the occupancy count

Probability of exceeding the capacity
Flight data provided by Eurocontrol (DDR2 semipublic database and Network manager data)

12 May 2016, 33,219 flights in 1,991 elementary sectors
Basic trajectory model

Starting and arrival airport

Many possible trajectories (sectors sequence)

Possible delay on each sequence

For flight $f$, we consider the set $\mathcal{R}_f = \{r_{f,1}, \ldots, r_{f,n}\}$ of probable trajectories (or scenarios). Each of the trajectories $r_{f,i} \in \mathcal{R}_f$ is associated with the probability $w_{f,i}$ that the flight will fly it.
Probability to be in a sector

Probability that flight $f$ is in sector $s$ at time $t$, if it follows trajectory $i$:

$$p_{f,s,t,r_{f,i}} := \Pr[\tau_{e,f,s,i} \leq t < \tau_{l,f,s,i}] = \Pr[\tau_{e,f,s,i} \leq t] - \Pr[\tau_{l,f,s,i} \leq t]$$

Distribution of entry time and leaving time is given

Probability that flight $f$ is in sector $s$ at time $t$:

$$p_{f,s,t} = \sum_{i=1}^{n} w_{f,i} p_{f,s,t,r_{f,i}}$$

One value for each combination of flight, time and sector: 95 billions values
Probabilistic occupancy count of a sector

Definition

We denote the probabilistic occupancy count of a sector \( s \) at time \( t \) as \( \Theta_{st} : \mathbb{N} \rightarrow [0, 1] \), which is a discrete pdf. For any number of flight, \( N \), the pdf \( \Theta_{st} \) will tell us what is the probability that \( N \) flights are in the sector \( s \) at time \( t \).

\[
\begin{align*}
\Theta_{st}(0) &\quad \text{probability that no flight is in sector } s \text{ at time } t \\
\Theta_{st}(1) &\quad \text{probability that one flight is in sector } s \text{ at time } t \\
& \quad \text{\ldots} \\
\Theta_{st}(m) &\quad \text{probability that } m \text{ flights are in sector } s \text{ at time } t
\end{align*}
\]

Convolution of the binomial distribution of all the flights [Irvine et al., 2011]

Exponential computational cost (with standard methods)
Probabilistic occupancy count of a sector

Dynamic programming scheme

If the probability that each flight is in sector $s$ at time $t$ is known, how can we compute the probabilistic occupancy count of this sector efficiently?

Dynamic programming technique: list the flights and add them recursively

$q(i,j)$: probability that $i$ flights among the $j$ first ones are in the sector

We need to compute the value $q(k,m)$ for all $k$

We have that:

$$q(i,j) = q(i,j-1) \left(1 - p_{f_j,s,t}\right)$$

$$+ q(i-1,j-1) \cdot p_{f_j,s,t}$$
Occupancy pdf

With this method, we are able to compute the occupancy count with a quadratic computational time.

We computed the full probabilistic distribution for the 1991 sectors at every minute (3 millions distributions).

Programs running in 3 hours.
Expected occupancy
Evolution over the day
Measures on sectors

Overload probability and Buffer ratio

The PDF is now known, what to do next?

Overload probability of a sector \( s \) at time \( t \), with \( C \) the maximal capacity of the sector:

\[
\omega_{st} = \Pr[\Theta_{st} > C_{st}] = \sum_{i > C_{st}} \Theta_{st}(i)
\]

Quadratic computational time

Buffer ratio of a sector at time \( t \):

\[
\varrho_{st} = \left( \mathbb{E}[\Theta_{st}] + \sqrt{\text{Var}[\Theta_{st}]} \right) / C_{st}
\]

The buffer ratio relies only on expectancy and variance, can be computed in linear time

Two measures for each sector at each time, 6 millions values
Outlook

- Quantification of sector congestion
- Criticality measures on flights
- Reduce network congestion
How to reduce the congestion?

We obtained measures of the congestion of the sectors

Which flights would be good candidates for STAMS or preventive actions (delaying, cancellation), to avoid congestion?

Aim: identify the flights which have the most impact on the network

Tools: measures of the sectors crossed by the flights
Congestion indices
Based on overload probability

1. Total congestion index, defined as the total sum of probabilities that a flight is in a sector, when it is a hotspot:

\[
TCI_f = \sum_{s \in S} \sum_{t \in T} p_{f,s,t} \omega_{st},
\]  \hspace{1cm} (1)

2. Average congestion index:

\[
ACI_f = \frac{1}{S_f} \sum_{s \in S} \sum_{t \in T} p_{f,s,t} \omega_{st},
\]  \hspace{1cm} (2)

where \( S_f \) is the total number of crossed sectors;

3. Max congestion index:

\[
MCI_f = \max_{s \in S} \sum_{t \in T} p_{f,s,t} \omega_{st};
\]  \hspace{1cm} (3)
Buffer Indices
Based on overload buffer ratio

1. Total buffer index, defined as the total sum of probabilities that a flight is in a dense sector:

\[ TBI_f = \sum_{s \in S} \sum_{t \in T} p_{f,s,t}Q_{st}, \]  \hspace{1cm} (1)

2. Average buffer index:

\[ ABI_f = \frac{1}{S_f} \sum_{s \in S} \sum_{t \in T} p_{f,s,t}Q_{st}, \]  \hspace{1cm} (2)

where \( S_f \) is the total number of crossed sectors;

3. Max buffer index:

\[ MBI_f = \max_{s \in S} \sum_{t \in T} p_{f,s,t}Q_{st}. \]  \hspace{1cm} (3)
Max indices

Flights with highest (0.5%) TCI
## Flights with highest indices

<table>
<thead>
<tr>
<th>Flight name</th>
<th>TCI</th>
<th>ACI</th>
<th>MCI</th>
<th>Dep.</th>
<th>Arr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*ELY5161</td>
<td>108.65</td>
<td>3.40</td>
<td>19.34</td>
<td>LLBG</td>
<td>LPPT</td>
</tr>
<tr>
<td>*LED2</td>
<td>104.85</td>
<td>14.98</td>
<td>64.56</td>
<td>ESOW</td>
<td>ENRK</td>
</tr>
<tr>
<td>IBE3317</td>
<td>87.62</td>
<td>3.13</td>
<td>14.00</td>
<td>LLBG</td>
<td>LEMD</td>
</tr>
<tr>
<td>ELY397</td>
<td>84.92</td>
<td>3.15</td>
<td>11.61</td>
<td>LLBG</td>
<td>LEMD</td>
</tr>
<tr>
<td>*N30678</td>
<td>83.22</td>
<td>5.55</td>
<td>16.01</td>
<td>LPAZ</td>
<td>LEIB</td>
</tr>
<tr>
<td>*N30701</td>
<td>79.85</td>
<td>4.99</td>
<td>14.57</td>
<td>LPAZ</td>
<td>LEIB</td>
</tr>
<tr>
<td>*RGR5541</td>
<td>77.86</td>
<td>2.78</td>
<td>6.76</td>
<td>LCRA</td>
<td>EGVN</td>
</tr>
<tr>
<td>*LED2</td>
<td>104.85</td>
<td>14.98</td>
<td>64.56</td>
<td>ESOW</td>
<td>ENRK</td>
</tr>
<tr>
<td>*JFA68G</td>
<td>36.54</td>
<td>9.13</td>
<td>15.97</td>
<td>LFQA</td>
<td>LFMT</td>
</tr>
<tr>
<td>OEFCS</td>
<td>32.42</td>
<td>8.10</td>
<td>23.76</td>
<td>LIBR</td>
<td>LIEO</td>
</tr>
<tr>
<td>*FMY8959</td>
<td>27.81</td>
<td>6.95</td>
<td>16.84</td>
<td>LFDB</td>
<td>LFMC</td>
</tr>
<tr>
<td>DMN26E</td>
<td>48.32</td>
<td>6.90</td>
<td>18.43</td>
<td>ESMQ</td>
<td>EDXF</td>
</tr>
<tr>
<td>*UIT462</td>
<td>48.29</td>
<td>6.90</td>
<td>12.08</td>
<td>ENTO</td>
<td>ESNO</td>
</tr>
<tr>
<td>CGMCP</td>
<td>40.70</td>
<td>6.78</td>
<td>22.21</td>
<td>BIKF</td>
<td>EIDW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight name</th>
<th>TCI</th>
<th>ACI</th>
<th>MCI</th>
<th>Dep.</th>
<th>Arr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*N30701</td>
<td>468.68</td>
<td>29.29</td>
<td>66.73</td>
<td>LPAZ</td>
<td>LEIB</td>
</tr>
<tr>
<td>*N30678</td>
<td>461.20</td>
<td>30.75</td>
<td>67.24</td>
<td>LPAZ</td>
<td>LEIB</td>
</tr>
<tr>
<td>*ELY5161</td>
<td>409.84</td>
<td>12.81</td>
<td>58.79</td>
<td>LLBG</td>
<td>LPPT</td>
</tr>
<tr>
<td>N8520K</td>
<td>394.82</td>
<td>30.37</td>
<td>67.18</td>
<td>LPAZ</td>
<td>LEVC</td>
</tr>
<tr>
<td>*RGR5541</td>
<td>380.69</td>
<td>13.60</td>
<td>30.58</td>
<td>LCRA</td>
<td>EGVN</td>
</tr>
<tr>
<td>RCH442</td>
<td>359.35</td>
<td>10.57</td>
<td>23.52</td>
<td>EGAA</td>
<td>LGSA</td>
</tr>
<tr>
<td>GAF229</td>
<td>353.45</td>
<td>17.67</td>
<td>56.86</td>
<td>LCPH</td>
<td>ETSA</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight name</th>
<th>TCI</th>
<th>ACI</th>
<th>MCI</th>
<th>Dep.</th>
<th>Arr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*FMY8959</td>
<td>132.97</td>
<td>33.24</td>
<td>50.93</td>
<td>LFDB</td>
<td>LFMC</td>
</tr>
<tr>
<td>*UIT462</td>
<td>231.19</td>
<td>33.03</td>
<td>47.77</td>
<td>ENTO</td>
<td>ESNO</td>
</tr>
<tr>
<td>*LED2</td>
<td>215.46</td>
<td>30.78</td>
<td>87.70</td>
<td>ESOW</td>
<td>ENRK</td>
</tr>
<tr>
<td>*N30678</td>
<td>461.20</td>
<td>30.75</td>
<td>67.24</td>
<td>LPAZ</td>
<td>LEIB</td>
</tr>
<tr>
<td>N8520K</td>
<td>394.82</td>
<td>30.37</td>
<td>67.18</td>
<td>LPAZ</td>
<td>LEVC</td>
</tr>
<tr>
<td>*JFA68G</td>
<td>118.02</td>
<td>29.51</td>
<td>41.44</td>
<td>LFQA</td>
<td>LFMT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight name</th>
<th>TCI</th>
<th>ACI</th>
<th>MCI</th>
<th>Dep.</th>
<th>Arr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*JTG855</td>
<td>204.77</td>
<td>18.62</td>
<td>102.55</td>
<td>EKCH</td>
<td>EKYT</td>
</tr>
<tr>
<td>*FEI32</td>
<td>242.51</td>
<td>22.05</td>
<td>89.79</td>
<td>BIRK</td>
<td>ESGP</td>
</tr>
<tr>
<td>*ENF02</td>
<td>177.83</td>
<td>25.40</td>
<td>88.38</td>
<td>LIEO</td>
<td>LIBD</td>
</tr>
<tr>
<td>*LED2</td>
<td>215.46</td>
<td>30.78</td>
<td>87.70</td>
<td>ESOW</td>
<td>ENRK</td>
</tr>
<tr>
<td>GW19960</td>
<td>161.55</td>
<td>8.97</td>
<td>81.66</td>
<td>EDDL</td>
<td>LDSP</td>
</tr>
<tr>
<td>PEG27</td>
<td>222.17</td>
<td>8.89</td>
<td>75.77</td>
<td>EGBB</td>
<td>LIRA</td>
</tr>
<tr>
<td>*RGR61XY</td>
<td>166.24</td>
<td>18.47</td>
<td>75.56</td>
<td>EGSS</td>
<td>LFAD</td>
</tr>
</tbody>
</table>
Outlook

• Quantification of sector congestion

• Criticality measures on flights

• Reduce network congestion
Ground holding policies

Delay of 15 minutes for 1% flights with highest indices
Result on flights of top .05% indices (blue)
Total overload probability

How to quantify the global congestion of the network?

Need for an easily computable metric

Sum for all the sectors and for all the times of the probabilities that a sector is overloaded at that time:

$$\Omega = \sum_{s \in S} \sum_{t \in T} \omega_{st}.$$ 

The value for the selected day is 105,409.06
## Effects of the actions

<table>
<thead>
<tr>
<th>Action type</th>
<th>$\Omega$ value</th>
<th>effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>105,409.06</td>
<td></td>
</tr>
<tr>
<td>Random ground holding</td>
<td>105,426.72</td>
<td>+0.01%</td>
</tr>
<tr>
<td>CI based ground holding</td>
<td>105,379.52</td>
<td>−0.3%</td>
</tr>
<tr>
<td>BI based ground holding</td>
<td>105,378.04</td>
<td>−0.3%</td>
</tr>
<tr>
<td>Random canceling</td>
<td>102,400.62</td>
<td>−2.85%</td>
</tr>
<tr>
<td>CI based canceling</td>
<td>98,881.99</td>
<td>−6.19%</td>
</tr>
<tr>
<td>BI based canceling</td>
<td>98,545.42</td>
<td>−6.51%</td>
</tr>
</tbody>
</table>

Actions on 1% on the flights. Ground holding for 14.4 minutes or canceling
Further work

• Design tailored strategies to minimise the congestion, i.e. work on the following problem:

\[ \min \sum_{s \in S} \sum_{t \in T} \omega_{st} \]

With delays or cancelations of flights as decision variables.

• Compare the strategies proposed with observed regulations on past data

• Include reactionnary delays in the model
Conclusion

Sector congestion:
• Probabilistic model for sector occupancy and quantification of overload probability
• Implementation of the model with quadratic computational complexity
• Measures for the congestion

Identification of critical flights:
• Indices on the contributions of flights to the network congestion

Search for solutions:
• Quantification for the network congestion and ways of reducing it

Questions?