Including Linear Holding in Air Traffic Flow Management for Flexible Delay Handling

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Outline

- Motivation & Background
- Trajectory optimization for linear holding
- Network ATFM model with linear holding
- Illustrative Examples
- Conclusions & Further Work
Motivation & Background

Linear holding concept

- Ground holding
- Airborne holding
- Linear holding

• Extra fuel consumption?
• Maximum holding time?
• Flexibility to absorb delay?
• Range/flexibility for implementation?
Motivation & Background

Direct operating cost

Not only fuel consumption but also time-related costs are considered

\[ CI = \frac{C_{\text{Time}}}{C_{\text{Fuel}}} \]

The higher the CI is, the more importance will be given to the trip time and the faster the optimal aircraft speed will be.
For all speeds between the equivalent speed and ECON, the fuel consumption will be the same or lower than the nominal while linear holding will be performed.
Motivation & Background

Potential of linear holding for ATFM under TBO

Aircraft already airborne flying slower could stop performing linear holding, accelerate to the nominal speed, and recover part of the delay, without burning extra fuel than initially scheduled.
Trajectory optimization for linear holding

Nominal trajectory generation

Airbus Performance Engineers Program (PEP)

Typical ATM partitioned speed segments

Planned route

Point-mass aircraft model

International Standard Atmosphere (ISA) Model

Aircraft performance model (PEP)

Flight profile

Lateral route

Cost function

Optimal control problem formulation

NLP (GAMS) CONOPT

Optimal 4D trajectory generation as initially scheduled


Trajectory optimization for linear holding

Linear holding trajectory

Maximizing total flight time

\[ \max J = \max \int_{t_0}^{t_f} dt \]

At no extra fuel than initially scheduled (nominal trajectory)

\[ s.t. \int_{t_0}^{t_f} FF(t)dt \leq F_{nom} \]

Fuel consumed within each flight segment

\[ M_{LH}^{(i)} = M_{nom}^{(i)}, i = CL_1, \ldots, DE_4 \]

Flight level and lateral route

\[ H_{LH}^{CL_4} = H_{nom}^{CL_4}, H_{LH}^{DE_1} = H_{nom}^{DE_1}, H_{LH}^{CR_{2m}} = H_{nom}^{CR_{2m}} \]

Note: Pre-tactical re-routing and flight level capping, as part of a possible ATFM negotiation, are out of the scope of this paper.
Network ATFM model with linear holding

Participation of airlines in the ATFM process

Using the method presented in section “Trajectory optimization for linear holding”, the linear holding trajectory can be generated based on the nominal trajectory.
Participation of airlines in the ATFM process

- The amount of delay absorption that linear holding can realize is constrained by the fuel consumption, which again is dependent on aircraft type, take-off mass, flight distance, etc.

- From the ATFM perspective, considering all these data would be a daunting work. Moreover, some of the airline's information is proprietary, such as aircraft mass and fuel consumption figures.

- From the airline perspective, they could have a clear view of all the information of their own flights, and thus have an intimate knowledge of the capability of each particular flight to absorb delays airborne.
Problem statement

- Linear holding (LH) possible, in addition to ground (GH) and air holding (AH).
- Delays are assigned at each designed position along the flight’s scheduled trajectory.
- Existing ATFM models would treat AH and LH (speed control) as the same. In this model we distinguish them and we need two sets of decision variables:

\[
x_{f,t}^j = \begin{cases} 
1, & \text{if flight } f \text{ departs from the position } j \text{ at time } t \\
0, & \text{otherwise} 
\end{cases}
\]

\[
y_{f,t}^j = \begin{cases} 
1, & \text{if flight } f \text{ arrives at the position } j \text{ at time } t \\
0, & \text{otherwise} 
\end{cases}
\]
Network ATFM model with linear holding

Current flight schedules

Network Strategic Tool (NEST) by EUROCONTROL

Demand Data Repository v2 (DDR2) by EUROCONTROL

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- Sector boundaries → Defined positions (and associated decision variables)
The (only) one more input that should be provided (by airlines) to the NM

Note: negative values appear in climb/descent because of the slightly trajectory differences caused from speed changes
Network ATFM model with linear holding

Model formulation

1) Objective function

Minimizing the total delay cost raised from ground holding (GH), airborne holding (AH) and linear holding (LH):

\[ \min(\text{cost}_{TD}) = \min(GH + \alpha AH + \beta LH) \]

Since \( TD = GH + AH + LH \), LH can be substituted:

\[ \min(\text{cost}_{TD}) = \min[\beta TD + (\alpha - \beta)AH + (1 - \beta)GH] \]

Taking account the fairness, the total delay is added with a coefficient: \( c_f = (t - r_{f}^{k})^{1+\epsilon} \)

The objective function can be arranged as:

\[ \min \sum_{f \in F} [\beta c_f h_f + (\alpha - \beta) a_f + (1 - \beta) g_f] \]

\[ c_f h_f = \sum_{t \in T^k_f, P(f,n_f)=k} (t - r_{f}^{k})^{1+\epsilon} y_{f,t}^{k} \]

\[ a_f = \sum_{t \in T^w_f, w \in P(f,i):1<i<n_f} t(x_{f,t}^{w} - y_{f,t}^{w}) \]

\[ g_f = \sum_{t \in T^k_f, P(f,1)=k} (t - r_{f}^{k}) x_{f,t}^{k} \]
Network ATFM model with linear holding

Model formulation

2) Flight operations constraints

\[
\sum_{t \in T_{f}^j} x_{f,t}^j = 1 \quad \forall f \in F, \forall j \in P_f, \quad \sum_{t \in T_{f}^j} y_{f,t}^j = 1 \quad \forall f \in F, \forall j \in P_f, \quad \sum_{t \in T_{f}^j} x_{f,t}^w + y_{f,t}^w \leq 1 \quad \forall f \in F, \forall w \in W, \forall t \in T_{f}^w, \\
\quad \forall t' \in T_{f}^w - [t, t + w^w],
\]

Each flight is assigned with only one “slot” (within a pre-defined solution search space) for **departing and arriving**, at each defined position along its scheduled flight trajectory

Maximum airborne holding time

\[
x_{f,t}^j + y_{f,t'}^j \leq 1 \quad \forall f \in F, \forall i \in [1, n_f - 1], \forall t \in T_{f}^j, \\
\quad P(f, i) = j, P(f, i + 1) = j', \quad \forall t'' \in T_{f}^j - [t + z_{f}^j,j', t + z_{f}^j,j' + v_{f}^j,j'],
\]

Linear holding upper bound, which is provided by airlines (set to 0 if such information is not available)

Minimum turnaround time for connective flights of arrival and departure

\[
x_{f,t}^k + y_{f,t'}^k \leq 1 \quad \forall (f, f') \in F, \forall t \in T_{f}^k, \\
\quad P(f', 1) = k, P(f, n_f) = k, \quad \forall t''' \in T_{f}^k \cap [t, t + q_{f,f'}^k].
\]
Network ATFM model with linear holding

Model formulation

3) Network capacity constraints
Traffic demand must not exceed the capacity of departure airport, arrival airport and en route sectors, respectively

\[ \sum_{f \in F: P(f,1) = k} x^k_{f,t} \leq D^k(\tau) \quad \forall k \in K, \forall t \in T(\tau), \forall \tau \in \mathcal{T} \]

\[ \sum_{f \in F: P(f,n_f) = k} y^k_{f,t} \leq A^k(\tau) \quad \forall k \in K, \forall t \in T(\tau), \forall \tau \in \mathcal{T} \]

\[ \sum_{f \in F: P(f,i) = w, P(f,i+1) = w'} (x^w_{f,t} - x^{w'}_{f,t}) \leq C^s(\tau) \quad \forall w \in s \subseteq S, \forall t \in T(\tau), \forall \tau \in \mathcal{T}, \forall i \in [1, n_f - 1] \]

4) Constraints on decision variables

\[ x^j_{f,t} \in [0, 1] \quad \forall f \in F, \forall j \in P_f, \forall t \in T^j_f \]

\[ y^j_{f,t} \in [0, 1] \quad \forall f \in F, \forall j \in P_f, \forall t \in T^j_f \]
Network ATFM model with linear holding

Model formulation

5) Constraints from updating delay assignment

Different from the stochastic dynamic models, full deterministic information (e.g., weather forecast) is assumed in this paper, such that it is feasible to realize the dynamic optimization by re-executing the model.

\[
x^j_{f,t}(\tau') + 1 = x^j_{f,t}(\tau') \quad \forall f \in F, \forall j \in P_f, \\
\quad \forall t \in T^j_f \cap [t_1, t_\sigma], \tau' \geq 1
\]

\[
y^j_{f,t}(\tau') + 1 = y^j_{f,t}(\tau') \quad \forall f \in F, \forall j \in P_f, \\
\quad \forall t \in T^j_f \cap [t_1, t_\sigma], \tau' \geq 1
\]

\[
x'^j_{f,t}(\tau') + 1 = x'^j_{f,t}(\tau') \quad \forall f \in F, \forall t \in T'^j_f, \\
\quad t_\sigma \in (t^j_f(\tau'), t'^j_f(\tau')), \tau' \geq 1
\]

\[
y'^j_{f,t}(\tau') + 1 = y'^j_{f,t}(\tau') \quad \forall f \in F, \forall t \in T'^j_f, \\
\quad t_\sigma \in (t^j_f(\tau'), t'^j_f(\tau')), \tau' \geq 1
\]

The assigned values, prior to the initial time of the new iteration, should be linked to the new decision variables.

New decision variables must start from the next position after finishing their current flight segment, because the current remaining segment might be not long enough to realize the amount of linear holding previously provided by airlines.
Illustrative Examples

Case of study setup

Network Strategic Tool (NEST) by EUROCONTROL

Scenario-1
- traffic flow (156 flights) destined at EHAM airport
- 1 arrival airport and 3 en route sectors with constrained capacity
- 06 AM – 12 AM, 24th Oct, 2016
- situation changes for better at 09 AM

Scenario-2
- traffic flow (2938 flights) across entire European airspace
- 6 airports and 78 en route sectors with constrained capacity
- 06 AM – 10 AM, 24th Oct, 2016
- situation changes for better/worse at 08 AM
Illustrative Examples

Some assumptions

• the discrete time interval was set to 1 min;

• $e = 0.05$ was selected as the fairness factor;

• the cost weights for AH and LH were, respectively, 1.2 and 0.8 with regard to the GH;

• the LH time bound was approximated as 20% [1] of the planned total trip time; and

• the delay updating can be initiated at once while flights can receive and execute immediately the latest delay assignment.

Illustrative Examples

Results of Scenario-1: Demand and Capacity

[Graphs showing traffic demand and capacity analysis for different time periods (h) for EHAM, EDDALL1, LFEKHRZIU, and LFEUXE.]
Illustrative Examples

Results of Scenario-1: Initial delay assignment

- Total Delay lower with LH since delay at different locations can be different \( \rightarrow \) useful if a flight crosses multiple regulations
- More aircraft delayed with LH because the weighted cost of LH is lower than GH and the LH time is bounded.

Initial assignment with no LH

Initial assignment by using LH

2421 min of Total Delay
86 delayed flights

2301 min of Total Delay
97 delayed flights
Illustrative Examples

Results of Scenario-1: Updated delay assignment

- **Updated assignment with no LH**
  - 1369 min of Total Delay
  - 66 delayed flights

- **Updated assignment by using LH**
  - 499 min of Total Delay
  - 38 delayed flights

**Aircraft on ground can depart at their earliest times**

**Less “unnecessary” delay on ground” AND aircraft in the air can speed up to recover delay (at no extra fuel cost)**
## Illustrative Examples

### Results of Scenario-1: Summary of results

<table>
<thead>
<tr>
<th>Cases of study</th>
<th>Total delay (min)</th>
<th>Delayed flights (a/c)</th>
<th>Av. total delay (min)</th>
<th>Total AH (min)</th>
<th>Total GH (min)</th>
</tr>
</thead>
<tbody>
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<td>2421</td>
<td>86</td>
<td>28.15</td>
<td>3</td>
<td>2418</td>
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<td>13.13</td>
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<td>370</td>
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</table>

<table>
<thead>
<tr>
<th>Cases of study</th>
<th>GH flights (a/c)</th>
<th>Av. GH (min)</th>
<th>Total LH (min)</th>
<th>LH flights (a/c)</th>
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<td>129</td>
<td>24</td>
<td>5.38</td>
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</tbody>
</table>
During the initial delay assignment, this particular flight is allocated with **41 mins of total delay** (arrival slot): **22 min GH + 19 mins LH**
Illustrative Examples

Results of Scenario-1: Summary of results

- Including linear holding means that more positions and periods can be used to absorb delays, rather than only at the departure airports prior to take-off. If multiple node constraints occur at the same time, separating delays at different places and periods could contribute to reducing the minimum delay required by multiple constraints.

- When the situation changes for better, benefiting from the shortening of ground holding (as substituted by linear holding), the departure time of a flight can be advanced. Once the delay assignment updated, less ground holding, and thus less total delay will be experienced.
Illustrative Examples

Results of Scenario-2: Demand and Capacity

- Case-0: Pre-regulation (scheduled flights)
- Case-1: Initial delay assignment for constrained capacity
- Case-2: Updating for improved capacity
- Case-3: Updating for reduced capacity

Note: Compared with Scenario-1, the effects of LH is not that remarkable in Scenario-2, since heavier network constraints, such as the departure capacity and the interaction among different flows of flights, are imposed in effect.
Illustrative Examples

Results of Scenario-2: Delay assignment

- When situation changes for better, aircraft already airborne are enabled to stop LH and accelerate immediately to meet a (potential) advanced controlled arrival time

- When situation changes for worse and the assigned delay is out of the LH upper bound, AH is performed. Still, most of the costly AH can be substituted by LH, thus lowering the total delay costs
Conclusions & Further Work

- The cost-based linear holding (LH) practice was included into the optimal allocation of ATFM delay, together with the commonly used ground and airborne holding measures.

- A trajectory generation method was presented, aiming at computing, per flight, the maximum linear holding realizable using the same fuel as the original nominal flight.

- This information was assumed to be computed and shared by the different airlines and it was then used to build a network ATFM model to optimally assign ATFM delays, in the scope of trajectory based operations (TBO).

- With LH, more space and periods in the network can be used to absorb delays, contributing to reducing the minimum system delay required from multiple constraints and the average delay per flight.

- Benefiting from the flexibility of linear holding, the ATFM performance of delay handling can be improved under uncertainty regardless of a better or worse situation change.
Conclusions & Further Work

Further work

- Fairness concern, especially the incentives of sharing (or reporting) the accurate information, from the perspective of different airlines
- Combined with other ATFM negotiation practices, such as the re-routting and flight level capping, to further enhance the potential of linear holding for flexible delay handling
- Allowing extra fuel than initially scheduled to be consumed, in line with the airline’s willing, such that the capability of delay absorption and recovery can be both improved
References


Thank you!
Any Questions?
Backup slides
Illustrative Examples

Results of Scenario-2: Demand and Capacity

- Case-0: Pre-regulation (scheduled flights)
- Case-1: Initial delay assignment for constrained capacity
- Case-2: Updating for improved capacity
- Case-3: Updating for reduced capacity

GA: \( GH + AH \)
GAL: \( GH + AH + LH \)

The inclusion of LH does not unnecessarily increase the capacity load. One may think that the reduced GH enabled by LH may lead to an increase of the airborne flights, somehow, aggravating en route traffic congestions.