An optimization model for assigning 4D-trajectories to flights under the TBO concept

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Abstract—The objective of this paper is to present a mathematical model that will contribute to the optimization and optimum configuration of the TBO concept. We develop a binary integer programming model whose aim is to assign a 4D-trajectory to each flight in order to optimize the efficiency of the ATM system. The model considers the preferred 4D-trajectory of all the flights in the pre-tactical planning phase and outputs an optimal pre-departure 4D-trajectory for each flight to be shared or negotiated with other stakeholders and subsequently managed throughout the flight. These output trajectories are obtained by minimizing the deviation (in terms of time delay, lateral and vertical deviation) from the original preferred trajectories. The particularities of this model are that it considers the complete 4D-trajectory for each flight as well as it incorporates the preferences and priorities of the ATM stakeholders. Some computational results are presented, which show that our optimization model has the ability to identify some trade-offs between the objectives of the stakeholders of the ATM system under the TBO concept. It can also provide the network manager with useful decision tools to choose a trajectory for each flight.

Keywords: integer programming; air traffic flow management; trajectory based operations.

I. INTRODUCTION

The European ATM Master Plan [1] and the SESAR Concept of Operations [2] identify the Trajectory Based Operations (TBO) concept as one of the cornerstones of the future Air Traffic Management (ATM) system. A central feature of the TBO concept is the consideration of the trade-offs existing between the needs and objectives of the individual airspace users to optimize their operations (user optimum) and the objective to ensure sufficient predictability and optimum performance of the ATM network as a whole (system optimum), without compromising flights’ safety. The development and implementation of the TBO concept requires the development of optimization models and algorithms that will allow pertinent decision makers and stakeholders to examine the trade-off between user and system optimum trajectories and to facilitate the definition of commonly accepted trajectories by all stakeholders. The objective of this paper is to present a mathematical model that will contribute to the optimization and optimum configuration of the TBO and to evaluate the performance of the TBO concept. This contributes to verify the viability of the TBO concept, to discern the major issues that need to be addressed, and whether, under which conditions, and to what extent the objectives, of flexibility of airspace users and predictability of the ATM system, can be achieved.

Several studies have been carried out in recent years to analyze the feasibility of the TBO concept. However, these studies focus more on the operational aspects of the problem and on the design of the single trajectory (see for instance Pleter et al. [3] and Wynnyk et al. [4]). Beginning with the seminal paper by Odoni [5], a great part of the academic/scientific Air Traffic Flow Management (ATFM) literature has focused exclusively on airport congestion, proposing solution approaches to alleviate such a situation. One of the first attempts to include en-route capacity restrictions in the ATFM problem was by Helme [6], who proposed a multi-commodity minimum-cost flow on a time-space network to assign airborne and ground delay to aggregate flows of flights. Lindsay et al. [7] formulated a disaggregate deterministic 0-1 integer programming model for assigning ground and airborne holding delay to individual flights in the presence of both airport and airspace capacity constraints. Bertsimas and Stock Patterson [8] presented a deterministic 0-1 integer programming model to solve a similar problem.

The first mathematical model that incorporates the flexibility of choosing the flight route among a set of possible alternatives, at least at a macroscopic level, is the model by Bertsimas and Stock Patterson [9]. This mathematical model is a dynamic, multi-commodity, integer network-flow. But its computational performance was not adequate for addressing problems encountered in realistic, very large-scale instances. A mixed 0-1 model is presented by Bertsimas et al. [10] which overcomes the latter computational limitation and is capable of addressing problems of a scale comparable to the two largest ATFM systems existing in the world, those that coordinate air traffic in the continental United States and in Western and
In 2012, Agustin et al. [11] presented a mathematical model that is closely related to the model of Bertsimas et al. [10]. This model adopts the same formulation of rerouting decisions and includes the possibility of cancelling flights as well as a cost for flights’ delays at intermediate waypoints along their flown route. Another research in this direction is by Churchill et al. [12], who proposed a mathematical model that focuses on air congestion at hot spots rather than modelling the whole ATM system.

All the above mathematical models only consider the 3D-trajectories of flights, meaning that aircraft trajectories do not define the altitude. One of the rare models, although not explicitly designed for 4D-trajectories, but which can be modified to capture the complete 4D-trajectories of flights is the one proposed by Sherali et al. [13]. This model prescribes a set of flight-plans – one for each flight – to be implemented. It seeks to minimize the delay and a fuel-cost-based objective function, subject to the constraints that each flight is assigned one of the designated flight-plans, and that the resulting set of flight-plans satisfies certain specified workload, safety, and equity criteria. In the scientific literature, no strategic models have been developed for the TBO concept. Kim and Hansen [14] investigated four alternative schemes of resource allocation within the Combined Trajectory Options Program.

In this paper, we develop a trajectory based mathematical model for the ATM system, which aims at optimizing the efficiency of the ATM system under TBO concept by assigning 4D-trajectories to flights based on the airspace users’ preferences and priorities, and the constraints of the ATM system. Thus, particular to our model is that, not only it considers the 4D-trajectories of aircraft, but it also incorporates the preferences and priorities of different ATM stakeholders.

The mathematical model is formulated as a binary optimization problem which considers the preferred 4D-trajectory of all the flights in the pre-tactical planning phase and outputs an optimal pre-departure 4D-trajectory for each flight to be shared or negotiated with other stakeholders and subsequently managed throughout the flight. These output trajectories are obtained by minimising the deviation (delay or re-routing) from the original preferred trajectories, in the presence of the constraints of the ATM system. The model is similar in spirit to the ones in [10, 11] and it aims at assigning a trajectory to each flight operating in the ATM system under the TBO concept. We define four objective functions, each representing a component of the ATM system that can be optimized within our model. First, the model seeks to minimize the total time deviation (delay) from the airspace users’ preferred trajectories. This time deviation is defined as a combination of airborne delay and ground holding delay (which should be less costly than airborne delay).

Secondly, the model considers the minimization of the total cost of deviation from the users preferred routes (lateral and vertical deviation). In other words, it minimizes the costs incurred when flights have to use arcs and flight levels that are not part of their preferred routes. The third objective of the model is to minimize the total airspace navigation service (ANS) charges of the system. Finally, the fourth objective considered in the model is to maximize the punctuality define in terms of the number of flights departing or arriving on time. Whilst the first three objectives are very important from the point of view of the airspace users, the latter is mainly important from the perspective of the airports. The constraints of the model are defined in order to ensure that each flight is assigned a unique 4D-trajectory and that the number of flights entering the airspace sectors, leaving and arriving to the airport, are kept under a controllable load for the air traffic controllers. The constraints also include the priorities of the airspace users. We carry out some computational experiments in order to validate the features of our model. Solution algorithms that will enable our model to solve larger and realistic problem instances will be in forthcoming research.

The remainder of this paper is organized as follows. In Section II, we discuss the preferences and priorities of the ATM stakeholders as considered in this research. The formulation of our mathematical model is presented in Section III. Some preliminary experiments to validate the model are presented in Section IV. Finally, Section V gives some concluding remarks and highlights the future works to be conducted in line with this paper.

II. Stakeholders’ Preferences and Priorities

Reference [15] reports the outcome of an ATM stakeholders’ workshop that was held in Brussels with the aim of identifying the needs, expectations, objectives, priorities and constraints of all the stakeholders in relation to the TBO concept.

A. Preferences

It is suggested in [15] that preferences can refer to any medium to absorb delays at the tactical level of ATM. This may result in a deviation from the preferred (4D) trajectories in terms of time (delay), flight altitude and lateral deviation (re-routing). In our mathematical model, the preferences of the ATM stakeholders are captured in two ways. First, the preferences of the airspace users are expressed in terms of how each of their flights should deviate (if need be) from its preferred 4D-trajectory i.e. a ranking between changes in flight level, lateral deviation and time deviation. In order to accommodate these preferences, our mathematical models will consider the ranking of the above types of trajectory deviation for each flight and define the costs parameters for the alternative trajectories accordingly. Second, our model considers four alternative objective functions to be optimized. Each of these objective functions targets a specific aspect of the efficiency of the ATM system that is to be optimized. Hence, the preferences of the ATM stakeholders can also be defined in terms of how well each of the objective functions achieves their individual interests.
B. Priorities

The prioritization scheme can be regarded as a framework which will provide the airspace users with the flexibility of managing and adapting their internal business models in a constrained environment [16]. To this end, SESAR has developed the concept of User-Driven Prioritization Process (UDPP), which enables the airspace users to optimize their flight schedules by managing time deviation (delays) during departure, en-route, and arrival. In other words, UDPP provides the airspace users with the capability of prioritizing their flights in case of capacity-constrained planning. The first step of UDPP is limited to departure slot swapping at tactical level [17]. UDPP Step 2 is foreseen beyond slot swapping and is considered at the planning phase. Our model considers prioritization at the planning phase. A SESAR research activity [19] proposes a prioritization mechanism (called the Fleet Delay Apportionment or FDA) where airspace users assign priority values to their flights and the system apportions delays to flights proportionally to their priority values. The allowable priority values are integers 1 through 9, with a value of 1 indicating the highest priority and of 9 indicating the lowest priority. We denote these priority values by $\tau_f$ for each flight $f$.

The overall idea of prioritization is for the airspace users to choose which of their flights should absorb more delay than the others in order to meet their business objectives. In other words, prioritization is a way around the First-Come-First-Served (FCFS) rule, which defines the maximum amount of time deviation (delay) in the preferred trajectory of each flight. Therefore, our mathematical model will capture the priorities through constraints which set the maximum amount of delay to be absorbed by each flight. More precisely, if we let $\sigma_f$ be the baseline delay (i.e., delay when prioritization is not allowed) of flight $f$ and $\Psi$ the set of all the airspace users, with an AU being $\varphi \in \Psi$, then the maximum amount of delay $\gamma_f$ to be assigned to flight $f$ under the FDA mechanism is calculated as [16]:

$$\gamma_f = \left(\frac{\tau_f \cdot \sigma_f}{\sum_{\varphi \in \Psi} \tau_f \cdot \sigma_f}\right) \cdot \xi_{\varphi},$$

where $\xi_{\varphi} = \sum_{f \in \Psi} \sigma_f / \varphi \in \Psi$ is the total amount of delay to be absorbed by airspace user $\varphi$ in the baseline delay, with $\mathcal{F}_{\varphi}$ being the fleet of AU $\varphi$. Note that in (1) the baseline delay is used as a reference in order to ensure equity between the airspace users i.e. to prevent the prioritization action of one airspace user from impacting negatively on the flight of others. In other words, the total amount of delays to be assigned to the fleet of an airspace user $\varphi \in \Psi$ will be equal to the total amount of delay that its fleet would have received in the baseline situation.

In what follows we summarize how the above priorities of the AUs will be incorporated and interact with the optimization models:

1. Simulation is run using the preferred 4D-trajectory of each flight to determine the total amount of delay to be absorbed by the system.
2. The “$\sum$ Baseline delays = $\sum$ UDPP delays” rule is applied to calculate how much delay $\xi_{\varphi}$ is to be assigned to the fleet of each AU.
3. The AUs re-distribute the amount of delay among their flights via priority credits/values $\tau_f \in \{1, \ldots, 9\}$ assigned to their flights.
4. A “prioritization constraint” (see (9)) is added to the optimization model for each flight $f$ according to the $\gamma_f$ values, and the optimization model is solved.

III. MATHEMATICAL FORMULATION

In our mathematical formulation, we represent the airspace as a network, which we define as a direct graph $\mathcal{G} = (\mathcal{N}; \mathcal{E})$ in the 2D-space, where $\mathcal{N}$ is the set of nodes (airports, waypoints on borders between two sectors) $\mathcal{E}$ is the set of arcs connecting the nodes. An arc can be traversed by flights at different flight levels and the set of all the flight levels will be denoted by $\mathcal{L}$. The time horizon considered will be discretized into time periods.

A. Data and notation

The data for our mathematical model is composed of a set of flights (herein denoted by $\mathcal{F}$), each having a preferred 4D-trajectory from the airport of origin to the airport of destination. The notation for the mathematical formulation is presented in the sequel:

- $\mathcal{K}$ is the set of airports.
- $\mathcal{S}$ is the set of en-route sectors.
- $\mathcal{T}$ is the set of (discretized) time periods.
- $d_f \in \mathcal{K}$ is the flight $f$ airport of departure.
- $a_f \in \mathcal{K}$ is the flight $f$ airport of destination.
- $\delta_f \in \mathbb{Z}_+$ is the maximum (allowed) variation of altitude for flight $f$, measured in terms of the number of flight levels.
- $\mathcal{L}_f \in \mathcal{T}$ is the scheduled departure time (STD) for flight $f$.
- $\tilde{t}_f \in \mathcal{T}$ is the scheduled arrival time (STA) for flight $f$.
- $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f)$ is a directed graph describing the possible flight paths (2D) of flight $f$. Without loss of generality, this graph can be considered acyclic.
- $\mathcal{L}_f$ is the set of feasible flight levels for the flight $f$.
- $\Delta_{f,n}^+$ and $\Delta_{f,n}^-$ are respectively the sets of outgoing and incoming arcs of node $n \in \mathcal{N}_f$.
- $\mathcal{T}_s$ the set of arcs entering en-route sector $s$. 

\( T'_f \equiv \begin{cases} T_f' ; \overline{T}_e' \end{cases} \) is the set of feasible time periods for flight \( f \) to fly on arc \( e \in E_f \).

\( I'_n \equiv \begin{cases} I'_n ; \overline{T}_n' \end{cases} \) is the set of feasible time periods for flight \( f \) to arrive at node \( n \in N_f \), note that \( I'_n = \bigcup e \in \overline{E}_n T'_e \).

\( \alpha_{p,e} \) and \( \alpha_{f,e} \) are respectively the maximum and minimum travel time (i.e. the number of time periods) for flight \( f \) on arc \( e \in E_f \).

\( D_k^e \) is the departure capacity of airport \( k \) at time period \( t \).

\( A_k^e \) is the arrival capacity of airport \( k \) at time period \( t \).

\( E_s^e \) is the capacity of the en-route sector \( s \) at time period \( t \).

\( C_{e,l} \) is the cost of flight \( f \) to use arc \( e \) at flight level \( l \).

\( R_s^e \) is the ANS route charge if flight \( f \) passes through the airspace sector \( s \).

\( C_f \) is the total cost incurred by flight \( f \) when using its preferred route (arcs and flight level).

### B. The decision variables

The decision variables for our model are defined similarly to those of the Bertsimas-Stock Patterson [9] model, which is also similar to the ones used in [10, 11]. These are binary variables which will define, for each flight, the position (arc being flown and the altitude) at each time period.

\[
x_{e,l}^f(t) = \begin{cases} 1 & \text{if flight } f \text{ is planned to travel on arc } e \in E_f \\
0 & \text{otherwise.}
\end{cases}
\]

It is assumed that a flight cannot travel on more than one arc during the same time period. We also assume that if a flight is schedule to be at a different altitude during the next time period, then the ascent or the descent can start during the current time period to ensure a smooth transition of flight levels. Note that an arc \( e \in E \) is defined by two nodes as \( e = (n,m) \) with \( n,m \in N \), which can be at two different flight levels. Therefore the flight level \( l \) in the definition of the decision variable \( x_{e,l}^f(t) \) refers to the flight level of \( m \) if \( e = (n,m) \) with \( n,m \in N \setminus \{d_f,a_f\} \), and when \( n = d_f \) (respectively \( m = a_f \)) \( l \) will refer to the altitude of the standard instrument departure route i.e. SID (respectively the standard arrival route i.e. STAR) on the path of the flight \( f \), which will both be denoted by \( l^0 \).

### C. The objective functions

The primary aim of this model is to minimize the total delay of the system. However, other objective functions that can also be optimized in the framework of this optimization model will also be defined.

1) **Time deviation**

The time deviation of a flight refers to how the time schedule in the assigned trajectory of the flight differs from the time schedule in its preferred trajectory i.e. the delay. In line with most ATFM literature [10, 11], we consider the time deviation as a combination of the costs of airborne delay and ground holding delay (which should be less costly than airborne delay). Therefore, the main objective function to be minimized is defined as:

\[
\sum_{f \in \mathcal{F}} \sum_{t \in T'_f} \sum_{e \in \overline{E}_t} C_{e,l}^f(t) \left( x_{e,l}^f(t) - x_{e,l}^f(t-1) \right) - \sum_{t \in T'_f} C_g(t) \left( x_{e,l}^f(t) - x_{e,l}^f(t-1) \right)
\]

where \( C_{e,l}^f(t) = (t - \overline{t}_f)^{1+\epsilon_1} \) represents the cost incurred if flight \( f \) is delayed by \( t - \overline{t}_f \) unit of time, \( t \in T'_f \), while \( C_g(t) = (t - \overline{t}_f)^{1+\epsilon_2} - (t - \overline{t}_f)^{1+\epsilon_2}, t \in T'_f \) is the cost reduction if part of the delay in incurred on the ground. \( \epsilon_1 \) and \( \epsilon_2 \) are two parameters chosen such that \( 0 < \epsilon_2 < \epsilon_1 < 1 \) in order to allow for airborne delays to be more penalized than ground holding delays. The first term in this objective function takes into account the cost of the total delay assigned to each flight, while the second term is a cost reduction when part of the delay is absorbed before take-off.

2) **Deviation from the users preferred routes**

Our model aims to assign to each flight a trajectory that is the closest possible to their preferred ones. This can be achieved by minimizing the total cost incurred when flights use arcs and flight levels that are not part of their preferred trajectories. Therefore, minimizing the total deviation from the flights’ preferred routes (3D space) will be equivalent to minimizing the following objective function.

\[
\sum_{f \in \mathcal{F}} \sum_{e \in \mathcal{E}_f} \sum_{l \in L_f} C_{e,l}^f \left( \overline{T}_{e,l}^f - C_f \right)
\]

It can be seen that if a flight travels through its preferred route (i.e. without incurring lateral or vertical deviation), then its cost of deviation (the term in the brackets) will be zero. This cost represents the cost of departing from the flight's preferred routes and the cost coefficients \( C_{e,l}^f \) are calculated by taking into
account the preferences of airspace users in terms of vertical and horizontal deviation.

3) Air navigation service charges

The objective here is to minimize the total cost of air navigation service (ANS) route charges for flights when assigning trajectories to flights. These charges are calculated as the sum of charges generated in the charging zones defined by States in the ECAC area [18]. The corresponding objective function to be minimized is then defined as follows:

$$\sum_{f \in F, e \in E_f, e \in E_f, i \in L_f} R^f_{e,i} x^f_{e,i} \left( \bar{T}^f_e \right)$$  (5).

4) Punctuality

In the ATM literature, punctuality is usually defined as the proportion of flights that exceed their scheduled departure and arrival times by more than 15 minutes (see [21] and [23]). Note that punctuality here is viewed from the perspective of airport managers. The AUs’ view of punctuality is already captured in the delay objective function (2). Moreover, Jacquillat et al. [22] point out that can be more challenging and expensive for an airport to hold an arriving aircraft in the air than a departing aircraft on the ground. Therefore, in this model, punctuality is optimized through the minimization of the number of flights that exceed their scheduled times of departure and arrival by no more than 15 minutes. Wherein, the arrival punctuality is weighted (by $\theta > 1$) to be more penalized than the departure punctuality. We denote $\theta$ to be the time period discretization of 15 minutes. The corresponding objective function to be maximized is then defined as follows:

$$\sum_{f \in F, e \in E_f, d_f, a_f} x^f_{e,d_f}(T^f_{e,d_f}) + \theta \left( \sum_{f \in F, e \in E_f, d_f, a_f} x^f_{e,a_f} \left( \bar{T}^f_{e,d_f} \right) \right)$$  (4).

D. The constraints

The constraints of the model are defined in order to ensure that each flight is assigned a single 4D-trajectory and that the number of flights entering the airspace sectors, leaving and arriving to the airport, are kept under a controllable load for the air traffic controllers.

i) The first set of constraints is concerned with the time connectivity of flights. These constraints are a direct implication of the variables definition. Indeed, if a flight has arrived at arc $e$ by time $t^*$, then $x^f_{e,i}(t)$ must have a value of 1 for all later time periods ($t \geq t^*$). They are stated as:

$$x^f_{e,i}(t - 1) - x^f_{e,i}(t) \leq 0 \quad \forall f \in F, e \in E_f, t \in T^f_{e,i}, l \in L_f.$$  (6)

The second group of constraints ensure that each aircraft $f$ flies a single route.

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n} \left( \bar{T}^f_{e,n} \right) \leq 1 \quad \forall f \in F, e \in E_f.$$  (7a)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n} \left( t - a_{f, e}^n \right) \leq \sum_{e' = 1}^{\Delta^f_{e,n}} x^f_{e,n} \left( t \right) \quad \forall f \in F, n \in N \setminus \{d_f, a_f\}, t \in T^f_{n}.$$  (7b)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n} \left( t - a_{f, e}^n \right) \geq \sum_{e' \in L_f} x^f_{e,n} \left( t \right) \quad \forall f \in F, n \in N \setminus \{d_f, a_f\}, t \in T^f_{n}.$$  (7c)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n}(t) \leq 1 \quad \forall f \in F, e \in E_f.$$  (7d)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n}(t) \leq 1 \quad \forall f \in F, e \in E_f.$$  (7e)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n}(t) \leq 1 \quad \forall f \in F, e \in E_f.$$  (7f)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n}(t) \leq 1 \quad \forall f \in F, e \in E_f.$$  (7g)

$$\sum_{e \in \Delta^f_{e,n}} x^f_{e,n}(t) \leq \sum_{e' \in L_f} x^f_{e,n} \left( \bar{T}^f_{e,n} \right) \forall f \in F,$$

$$n \in N \setminus \{d_f, a_f\}, t \in T^f_{n}.$$  (7h)

More specifically, constraints (7a) ensure that flight level is constant (no changes) while aircraft is flying a specific arc. Constraints (7b) and (7c) represent the connectivity between arcs (sectors). Constraints (7b) state that a flight must arrive at one of the subsequent arcs (sectors) by at most $a_{f, e}^n$ time units (the maximum possible) after traveling through the preceding arc (sector). Constraints (7c) stipulate that a flight cannot arrive at an arc $e$ (outgoing arc of node $n$) by time $t$ if it has not arrived at one of the preceding arcs by time $t - a_{f, e}^n$. In other words, a flight cannot enter the next arc (sector) on its path until it has spent at least $a_{f, e}^n$ time units (the minimum
iii) The third group of constraints are the capacity constraints. These constraints respectively ensure that the departure capacity, the arrival capacity and the en-route sectors capacity cannot be exceeded, thus keeping the load of the air navigation service provider (ANSP) under a controllable level.

\[
\begin{align*}
\sum_{f \in F, k \in K} \left( x_{e,f}^0(t) - x_{e,f}^0(t-1) \right) & \leq D_k^t, \\
\forall t & \in T, k \in K. \\
\sum_{f \in F, k \in K} \left( x_{e,f}^0(t) - x_{e,f}^0(t-1) \right) & \leq A_k^t, \\
\forall t & \in T, k \in K. \\
\sum_{f \in F, s \in S} \left( x_{e,s}^f(t) - x_{e,s}^f(t-1) \right) & \leq E_s^t, \\
\forall t & \in T, s \in S.
\end{align*}
\]

Constraints (8a) ensure that the number of flight departures from airport \( k \) at time period \( t \) does not exceed its departure capacity at that time period. Constraints (8b) play the same role for arrivals at airport \( k \) at time period. Finally, constraints (8c) ensure that the number of flights in the en-route sector \( s \) at time period \( t \) is not more than its capacity at that time period. Observe that, sector capacity is measured as the number of flights entering in sector \( s \) at time period.

iv) Prioritization constraints. These constraints will ensure that the priorities of the airspace users, as defined in Section II, are taken into account. More specifically, they will ensure that the total delay assigned to each flight does not exceed the maximum amount of delay \( y_f \) to be absorbed by that flight as per the AU’s priority point.

\[
x_{e',f}^0(t) + x_{e',f}^0(t^\prime - t_f) + y_f \leq 0, \\
\forall f \in F, e' \in \Delta_{f,df}, e \in \Delta_{f,df}, t \in I_{df},
\]

where \( y_f \) is calculated as shown in (1).

v) Additional constraints can be included in order to represent airspace users’ specific requirements or constraints. For instance in the scientific literature [10, 11] it is common to include constraints representing connectivity between flights. These constraints handle the cases in which a flight is continued, i.e., the aircraft is scheduled to perform a subsequent flight within some user-specified time interval. Similar constraints can easily be added to our model.

E. Complexity analysis

The total number of variables as well as the number of constraints for the above model are in the order of \( \mathcal{O}(|F|, |T|, |N|^2, |L|) \). On the other hand, there are about 30 000 flights crossing the European airspace on a typical summer day. There are also nearly 700 airspace sectors and 600 airports. Therefore, with a 5 minutes time discretization of a 24 hour time horizon, and flight levels between FL0 and FL600, the model can be expected to have variables and constraints in the order of \( 10^{18} \).

In order to deal with such a large problem, a pre-processing phase will first be implemented to reduce the number of variables. For example, each flight has a very restricted feasible network. Therefore, the pre-processing phase will focus on eliminating those variables that are not in the feasible network of each flight. Furthermore, we aim to implement a column generation algorithm for solving this problem. This approach allows to consider only a small feasible set of trajectories for each flight to start with, and more feasible trajectories will be added progressively if needed.
The mathematical model presented in section III is a binary integer programming model. Solving such models for a realistic ATM system with a large number of variables, for example the entire ECAC area, necessitates the development of sophisticated solution approaches. The development of such solution methods will be the next step in our research. However, in order to test our mathematical model, we consider a small and tractable example of an airspace with 4 sectors, 3 airports and 20 flights. Figure 1 illustrates the projection of the network that we considered for this computational experiment unto a two dimensional space. The full network includes different flight levels for the arcs and the sectors. The four airspace sectors are {I, II, III, IV} and the airports are A, B and C. Here the nodes are airports and waypoints in between two sectors {A, B, C, a, b, c, d, e, f, g}. Note that, each arc can be traversed at different flight levels. The numbers on the arcs in Figure 1 represent the cost incurred by each flight to traverse these arcs. Indeed these costs are not the same for the different flight levels.

In total there are 16 arcs for each flight level in the network, which are not all feasible for all the flights. The set of arcs feasible for each flight depending on their origin-destination pair is provided in Table 1. We consider 6 possible flight levels. The time horizon is from 6am to 6pm with a table: Arc feasible for each origin-destination pair

<table>
<thead>
<tr>
<th>Origin Destination</th>
<th>Feasible arcs</th>
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<tbody>
<tr>
<td>A-B or B-A</td>
<td>(A,c), (c,B), (A,b), (b,d), (d,B)</td>
</tr>
<tr>
<td>A-C or C-A</td>
<td>(A,a), (A,b), (a,e), (a,d), (b,e), (b,d), (d,f), (d,g), (e,C), (f,C), (g,C)</td>
</tr>
<tr>
<td>B-C or C-B</td>
<td>(B,f), (B,g), (f,C), (g,C)</td>
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</tbody>
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<thead>
<tr>
<th>Flights</th>
<th>Departure</th>
<th>Arrival</th>
<th>STD</th>
<th>STA</th>
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Table 2: Some parameter for the experiments
discretization of 30 minutes, making a total of 25 time periods to be considered. In Table 2, we describe few important parameters for the flights in this example. Namely, Table 2 gives the airports of departure and arrival of each flight as well as their scheduled time of departure (STD) and scheduled time of arrival (STA). Note that these times are given in terms of time periods instead of clock time. The departure times for the flights are generated randomly, while the arrival times are assigned randomly and considered to reflect the preferences of the airspace users. The flight levels admissible for each flight are also generated randomly.

For this example, we conducted two experiments. The first experiment consisted of solving the optimization model, where the objective function is (3) subject to the constraints (6)-(9). In other words, we find the best assignment of 4D-trajectories to flights that minimizes the total cost of deviating from the users’ preferred routes. For this example we defined the cost in terms of the duration of the journey. The capacities of the airports (arrival and departure) and en-route sectors have been set to 3 for all time periods. We solved this model directly using the CPLEX MIP solver [20]. The results of this first test are reported in Figure 3. This figure shows the scheduling of each flight by means of the time at which the flight leaves the airport, the amount of time spent in each sector as well as the time of arrival of each flight. It can be seen in this figure that all the flight use the shortest path available to them although they do not leave or arrive on time. In addition, these results give an idea of how the flights should be scheduled against each other (in terms of time of departure, time of arrival) if the system seeks to minimize the total deviation from the users’ preferred routes.

For the second experiment, we change the objective function to minimize the total time deviation (or delay) from the users’ preferred trajectories i.e. we now minimize objective function (2) subject to the constraints (6)-(9). All the other parameters were kept as previously. The results for this second experiment are shown in Figure 2. The information shown in this figure are the same as the ones shown in Figure 3. However, it appears here that most flights are able to stick to their schedule in terms of departure and arrival. Few of them are not able to keep their preferred trajectories because of the capacity restriction of the system. For example, Flight 2 was scheduled to leave airport A during time period 4 and arrive at airport C during time period 14. However, at time period 4, there are already three flights (3, 4 and 7) taking off at this airport, so Flight 2 is delayed by one time period. But it arrives to its destination on time after being assigned a shorter route than its initial preferred route. This experiment shows that our model is capable of assigning 4D-trajectory to flights while minimizing the total time deviation of the system.

From these numerical experiments, we can note that the output model can differ from one another depending on the objective function that is being minimized. For example, when minimizing the total cost of deviation from the users’ preferred routes, it can be seen that flights 2, 3 and 4 are scheduled to leave and arrive at the same time. However, their times of departure and arrival are different when minimizing.
the total time deviation from the users’ preferred trajectories of the system. Furthermore, each assignment of trajectory in our model has the ability to optimize a specific aspect of the ATM system and achieve to some extend the preferences of different ATM stakeholders. This shows that our optimization model is capable to identify some trade-offs between the objectives of the stakeholders of the ATM system under the TBO concept and to provide the network manager with useful decision tools to assign a trajectory to each flight.

V. CONCLUSION

In this paper, we have presented a trajectory based mathematical models for the ATM system. This model aims at optimising the efficiency of the ATM system under the TBO concept by assigning 4D-trajectories to flights based on the airspace users’ preferences and priorities, and the constraints of the ATM system. The particular aspect of our model is that, not only it considers the complete 4D-trajectories of aircraft, but it also incorporates the preferences and priorities of the ATM stakeholders. The model is formulated as a binary optimization problem which considers the preferred 4D-trajectory of all the flights in the pre-tactical planning phase and outputs an optimal pre-departure 4D-trajectory for each flight to be shared or negotiated with other stakeholders and subsequently managed throughout the flight. These output trajectories are obtained by minimising the deviation (delay or re-routing) from the original preferred trajectories, ANS route charges as well as maximizing the punctuality of the system in the presence of the constraints of the ATM system. We carried out some computational experiments in order to validate the features of our model.

The proposed model can be used by the network manager to choose a trajectory for each flight under the TBO concept. This model includes various objective functions, each of which represents an aspect of efficiency of the ATM system that to be optimized. It appears that there exist some trade-offs between these objectives. Therefore, depending on the objective of the network manager different trajectories can be assigned to each flight. A further investigation of the trade-offs between these objectives will be carried out in the next steps of this research, which is to develop and implement sophisticated solution algorithms to solve this optimization model for the ATM system of the entire ECAC area. We aim to adopt a multi-objective optimization approach, wherein we consider all the four objective functions (2), (3), (4) and (5). The multi-objective optimization approach will further contribute to achieving the preferences of the ATM stakeholders.

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